Quick and Accurate Strategy for Calculating the Solutions of the Photovoltaic Single-Diode Model Equation

Fco. Javier Toledo, M. Victoria Herranz, José Manuel Blanes, and Vicente Galiano

***Abstract*—The photovoltaic (PV) single-diode model is the most widely used to characterize the behavior of a PV panel because it combines high precision with moderate difficulty. Lots of methods to obtain the model parameters use optimization techniques that require the resolution of the characteristic equation thousands of times; therefore, it is essential to calculate its solutions accurately but also in the shortest possible time. The objective of this article is to describe a new numerical strategy to solve the characteristic equation in a simple, fast and precise way. The main idea is based on a reparameterization of the Lambert equation which is closely related with the infinite power tower, and some well-known bounds of this tower will be used as seeds of numerical methods. This strategy is powerful for certain "small" values of the Lambert W function argument, but it is combined with another re-expression of the Lambert equation for the remaining "large" values. The proposed numerical strategy is so precise in very few iterations that it can be suitably transformed into an explicit formula. The results obtained have been compared with some of the best options in the literature and experimental results prove the power of the proposed methodology.**

***Index Terms*—Characteristic equation of a photovoltaic (PV) panel, hyperexponentiation, Lambert W function, photovoltaic single-diode model (SDM), tetration.**

I. Introduction

IT IS well known that the behavior of a photovoltaic (PV) solar panel can be described under certain luminosity conditions by the PV single-diode model (SDM) [1] (see Fig. 1) whose equation is given by

$$I = I_{\text{ph}} - I_{\text{sat}} \left(\exp\left(\frac{V + I\,R_s}{a} \right) - 1 \right) - \frac{V + IR_s}{R_{sh}} \tag{1}$$

where I is the panel current, V is the panel voltage, I_{ph} is the panel photocurrent and I_{sat} is the panel diode saturation, R_s is the panel series resistance, and R_{sh} is the panel shunt resistance. The denominator in the exponential argument is $a = n_s n V_T$ where n_s is the number of cells in series, n is the diode ideality factor, and $V_T = k/q\,T$ is the so-called thermal voltage, where T is the temperature, k is the Boltzmann's constant and q is the electron charge.

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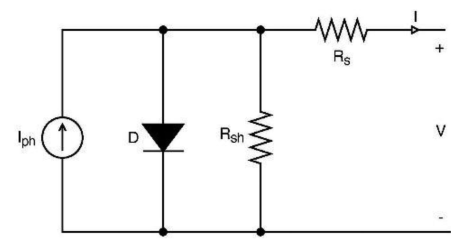

Fig. 1. PV module SDM electrical circuit.

The SDM is one of the most widely used models in the literature to characterize the behavior of a PV panel because it combines high precision with moderate difficulty. However, the transcendent nature of its characteristic equation (1) means that, even today, it continues to be subject of study. There are more complete models that normally include more than one diode [2] which are able to describe a larger range of situations like the behavior of the panel under low illumination and the effects of grain boundaries and leakage current [2], but it is significantly more difficult to deal with the corresponding mathematical equations which include more exponential terms.

Equation (1) defines the current I as an implicit function of the variable V which is indefinite differentiable along all the real line [3]. Nevertheless, it is possible to write I "explicitly" in terms of V [4] as

$$\begin{aligned} I = {} & \frac{1}{R_{\text{sh}} + R_s} \left(R_{\text{sh}} \left(I_{\text{ph}} + I_{\text{sat}} \right) - V \right) \\ & - \frac{a}{R_s} W_0 \left(\frac{I_{\text{sat}}\, R_{\text{sh}}\, R_s}{a \left(R_{sh} + R_s \right)} exp \left(\frac{R_{\text{sh}}}{a \left(R_{\text{sh}} + R_s \right)} \left(R_s \left(I_{\text{ph}} + I_{\text{sat}} \right) + V \right) \right) \right) \end{aligned} \tag{2}$$

where W_0 is the principal branch of the Lambert W function, that is, the inverse of the function $f(x) = xe^x$ in the interval $[-1, +\infty[$ [5]. The word "explicitly" is highlighted with quotes in the previous paragraph because it is worth noting that the Lambert W function is itself an implicit function and then, given V, the computation of the corresponding I with (2) needs numerical calculus [6], [7].

Also, V can be written "explicitly" in terms of I using again the Lambert W function as

$$V = R_{\mathrm{sh}}\left(I_{\mathrm{ph}} + I_{\mathrm{sat}}\right) - \left(R_{\mathrm{sh}} + R_s\right) I - aW_0\left(\frac{I_{\mathrm{sat}} R_{sh}}{a} \exp\left(\frac{R_{\mathrm{sh}}}{a}\left(I_{\mathrm{ph}} + I_{\mathrm{sat}} - I\right)\right)\right). \quad (3)$$

The procedure of obtaining I from a given V will be called *I-approach*, and that of obtaining V from a given I will be called *V-approach*.

A huge amount of works in the literature use the alternative expressions (2) and (3), which is an indicator of their importance. A small selection of these works is shown below, they address different topics related to the theoretical and applied study of the SDM. For example, [8]–[12] are devoted to extract the five SDM parameters using different techniques which depend on the type of data available. Jain and Kapoor [13] presents a method to study different parameters of organic solar cells. Ortiz-Conde *et al.* [14] presents exact closed form solutions to express the forward current-voltage characteristics containing all possible combinations of series and shunt parasitic resistances. Petrone *et al.* [15] introduces a new model of PV fields which allows the simulation of a PV generator whose subsections work under different solar irradiation values and/or at different temperatures. Piliougine *et al.* [16] analyses a selection of explicit methods with the aim of testing their capability to detect degradation in PV modules. In [17], a reformulation of the SDM allows to derive simplified formulae to calculate the maximum power points (MPP) of a PV string operating under different irradiance levels. Fathabadi [18] proposes a technique for tracking the MPP of PV modules connected in various configurations. In [19], a set of mathematical equations are derived to estimate the power peaks of overlapped bypass diodes PV arrays. In [20], the multiple MPPs that appear on the *P–V* characteristic of an array in partial shading conditions are derived using only datasheet information. In [21], by using piecewise curve-fitting method, an MPP detection method for PV module is presented.

All the methodologies used in the previous papers need to obtain the images of W_0 using a numerical method, an approximate formula, or a mathematical program that has already implemented a procedure for its calculation.

It is very important to be able to calculate the images of the Lambert W function as quickly and accurately as possible since many of the existing procedures for obtaining the SDM parameters require performing thousands, even millions, of times this calculus. Indeed, a real and maybe the most important application of the results of this article is to implement the proposed methodology to calculate the solutions of the SDM equation in an optimization procedure, such as the least squares method, to obtain the parameters of the SDM by fitting a set of voltage-current experimental points measured from a PV module. This optimization procedure can be used as an additional step to any known parameter extraction method as done in [22].

The main objective of this article is to provide a simple, fast and accurate strategy to calculate the solutions of the SDM equation by means of an alternative way of obtaining the images of the principal branch of the Lambert W function. In Section II, the mathematical aspects of the new strategy are motivated and explained based on suitable reparameterizations of the Lambert equation depending on the size of the function arguments. Section III provides and justifies the selection of the seeds chosen for the numerical methods which lead to find the roots of the functions suggested in Section II. In Section IV, the algorithms which give the solutions of the SDM equation are completely described by means of both flowcharts. Section V exposes the experimental results and Section VI shows the main conclusions of the article. Paper finishes with Section VII which includes the relevant references used in the manuscript.

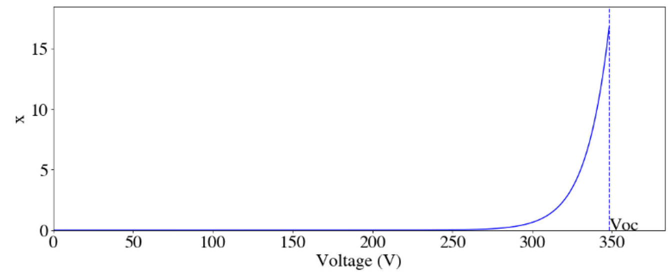

Fig. 2. Graph of (5) for the SDM parameters: I_{ph}=15.88A; I_{sat}=7.44·10^{-10} A; $R_s = 2.04\ \Omega$; $R_{\mathrm{sh}} = 425.2\ \Omega$; and $a = 14.67$.

II. New Strategy to Solve the SDM Equation

As it has been explained in Section I, the solutions of the SDM equation can be obtained through the computation of the images, $W_0(x)$, of the principal branch of the Lambert W function. Recall that, calculating $W_0(x)$ for $x \in [-\frac{1}{e}, +\infty[$ is nothing else but finding the unique solution w in $[-1, +\infty[$ of the equation

$$we^w = x. \quad (4)$$

In the SDM (2), the argument of the function W_0 is

$$x = \frac{I_{\mathrm{sat}}\ R_{\mathrm{sh}}\ R_s}{a\left(R_{\mathrm{sh}} + R_s\right)} \exp\left(\frac{R_{\mathrm{sh}}}{a\left(R_{\mathrm{sh}} + R_s\right)}\left(R_s\left(I_{\mathrm{ph}} + I_{\mathrm{sat}}\right) + V\right)\right). \quad (5)$$

In real physical conditions, all the parameters in (5) are positive, then x in (5) is a strictly increasing function of V (see Fig. 2). If we take in mind that R_{sh} is usually a relatively large number with respect to R_s which is in turn small close to zero, the function of V given in (5) should be small, indeed near to zero for usual parameters, that is, parameters obtained under normal environmental conditions and nonextreme degraded panels, for a large range of V values, the more the V closer to zero, which determines in this case the calculation of $W_0(x)$.

It can be observed in the graph of Fig. 2 that most of the values of x are very small, indeed close to zero, compared with the value at the open circuit voltage V_{oc} which is the voltage corresponding to zero current.

The idea proposed in this case is based on rewriting (4) so that the resulting equation has certain properties that are especially beneficial in calculating its roots for values of x in the interval $[-1/e, e[$, but, in particular, for positive x values close to zero; observe that in the previous interval, the values in $[-1/e, 0]$ correspond to negative parameters in the SDM equation.

Equation (4) can be written in the form

$$z - G^z = 0 \quad (6)$$

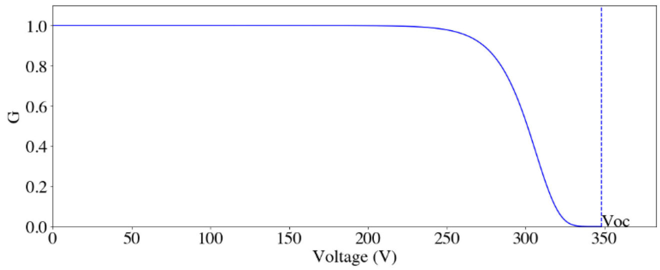

Fig. 3. Graph of $G = e^{-x}$ as a function of V for the same SDM parameters of Fig. 2.

where $G = e^{-x} \in]0, e^{\frac{1}{e}}]$, and $z = \frac{w}{x}$ if $x \neq 0$. In the particular case $x = 0$ the solution of (4) is just $w = 0$. Observe that a solution of (6) is indeed a fixed point of function G^z.

Define the function $f_G : R \to R$ given by

$$f_G(z) = z - G^z \tag{7}$$

for each $G \in]0, e^{1/e}]$, where R denotes the whole real line. f_G is a continuous and indefinitely differentiable function along all R and satisfies the following properties.

1) If $0 < G \leq 1$, f_G has a unique root, z_0 in the interval $]0, 1]$. In the particular case $G = 1$, $z_0 = 1$.
2) If $1 < G \leq e^{1/e}$, f_G has a unique root z_0 in the interval $]1, e]$. In the particular case $G = e^{1/e}$, $z_0 = e$. It should be said that in the case where $1 < G \leq e^{1/e}$, f_G has another root in the interval $]e, +\infty[$ which is related with the images of the negative branch of the Lambert W function.

It can be observed that, if $G > e^{1/e}$, $f_G < 0$ and, consequently, it has no roots.

Therefore, given $x_0 \in [-\frac{1}{e}, +\infty[$, if z_0 is the root of the function f_G in $]0, e]$, with $G = e^{-x_0}$, then $w_0 = z_0 \cdot x_0$ is the solution of (4) in $[-1, +\infty[$ and, then

$$W_0(x_0) = x_0 \cdot z_0. \tag{8}$$

Using (8), (2) can be rewritten as

$$I = \frac{1}{R_{\text{sh}} + R_s}\left(R_{\text{sh}}\left(I_{\text{ph}} + I_{\text{sat}}\right) - V\right) - \frac{a}{R_s} x_0 z_0$$

where x_0 is the argument of the Lambert W function given in (5) for a certain voltage V_0.

es If the argument of the function W_0 in (2) is close to zero, the value of G will be then close to 1 (see Fig. 3).

The calculation of the roots of the function f_G for values of G close to 1 is extremely fast and precise thanks, in particular, to the known bounds, indeed good approximations, of the roots of the function f_G through the Infinite Power Tower which will be introduced in Section III.

In general, one of the advantages of solving (6), or of finding the roots of f_G, compared to solving (4), could be that the solutions of (6) are found in the bounded interval $]0, e]$, while the solutions of (4) lie in the unbounded interval $[-1, +\infty[$. However, although the calculation of the roots of f_G has no theoretical limitations, it could have computational weaknesses when the values of G are "extremely" close to zero, which corresponds precisely to "very" large values of x since $G = e^{-x}$. In this case, there are different alternatives in the literature to calculate the images of W_0 [23]. One of the most used techniques to obtain these images is through the asymptotic development of the Lambert W function [5], [24], [25] given by

$$L1 - L2 + \frac{L2}{L1} + \frac{L2\,(-2 + L2)}{2(L1)^2} + \frac{L2\left(6 - 9L2 + 2(L2)^2\right)}{6(L1)^3} + \frac{L2\left(-12 + 36L2 - 22(L2)^2 + 3(L2)^3\right)}{12(L1)^4} + \ldots \tag{9}$$

where $L1 = \ln(x)$ and $L2 = \ln(\ln(x))$.

This formula is suitable for "large" values of x while the number of terms (summands) depends on the level of accuracy needed. In [26], it is found that a truncation of this formula works well in the examples proposed there, however, (9) can also have computational problems for very large x values that exceed the double-precision maximum floating point numerical representation (returning overloads errors, infinite (Inf) or not a number (NaN) in different programming languages). To avoid this problem in the case of the SDM equations, in [26] it is proposed an ad hoc solution.

Below it is described a different procedure, already used in the literature, to be used for "large" values of x, at least $x > e$, which corresponds to "small" values of G, $G < 1/e^e$.

Taking logarithms in the equation $we^w = x$, whenever $x > 0$, which implies at the same time that $w > 0$, it is obtained the equation

$$\ln w + w - \ln x = 0.$$

For each $x > 0$, define the function

$$h_x(w) = \ln w + w - \ln x.$$

It is very easy to prove but also very well-known that h_x is a continuous, indefinitely differentiable and strictly increasing function in $]0, +\infty[$, with $\lim_{w \to 0^+} h_x(w) = -\infty$ and $\lim_{w \to +\infty} h_x(w) = +\infty$, then, h_x has a unique root in $]0, +\infty[$ which is the unique solution in this interval of the equation $w\,e^w = x$, and, then, it is the image of the Lambert W function.

The argument of the function W_0 in (3) is given by

$$x = \frac{I_{\text{sat}} R_{\text{sh}}}{a} \exp\left(\frac{R_{\text{sh}}}{a}\left(I_{\text{ph}} + I_{\text{sat}} - I\right)\right). \tag{10}$$

For positive values of the parameters, the function of I given in (10) is strictly decreasing. Since R_{sh} is usually a large value, this function will tend to reach very large values, the more the I is closer to 0, which suggests that in this case, the appropriate function for the calculation of $W_0(x)$ could be h_x because logarithms reduce the size of very large values to more computationally manageable values. The values of x in Fig. 4 are

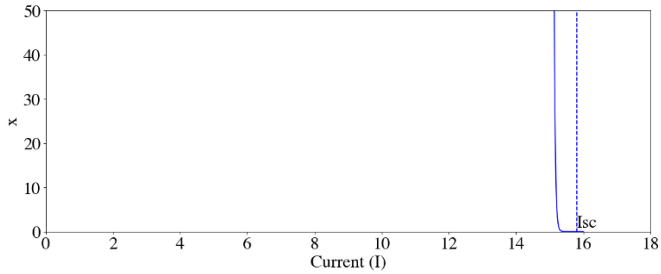

Fig. 4. Graph of (10) for the SDM parameters of Fig. 2.

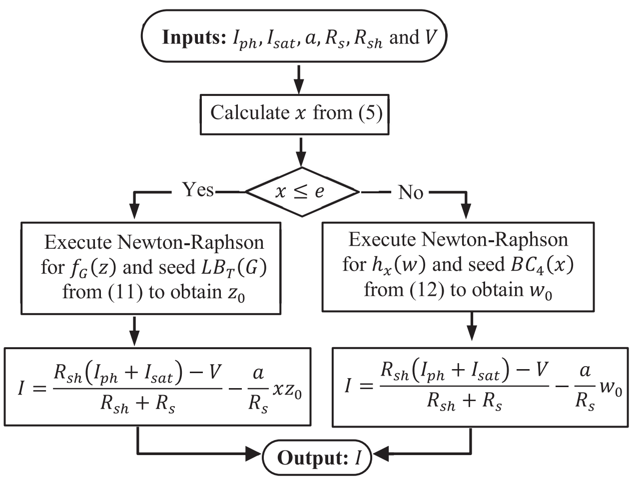

Fig. 5. Flowchart *I*-approach.

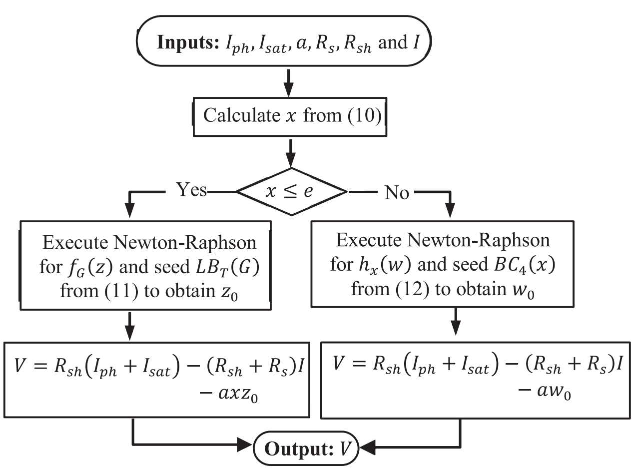

Fig. 6. Flowchart *V*-approach.

so large close to $I = 0$ that it is not possible to rescale suitably the graph. The value I_{sc} represents the short-circuit current, that is, the current for zero voltage.

An interesting approach to compute V in terms of I avoiding the possible extreme large values of x in (10) is given in [27], where (3) (or (1)) is rewritten in terms of $\bar{x} = \ln x$ scaping the use of x.

III. Calculation of the Roots of Functions $\boldsymbol{f_G}$ and $\boldsymbol{h_x}$

A. Calculation of the Roots of $\boldsymbol{f_G}$

To get a good seed for calculating the roots of the function f_G defined in (7), its relationship with the well-known Hyper-power function is going to be used. If z_0 is a root of f_G, then $z_0 = G^{z_0}$, therefore

$$z_0 = G^{(G^{z_0})} = G^{\left(G^{\left(\cdot^{\cdot^{\cdot^{(G^{z_0})}}}\right)}\right)}.$$

The sequence T_n recursively defined by $T_0 = 1$ and $T_n = G^{T_{n-1}}$ for $n = 1, 2, \ldots$, is called Hyperpower of base G and height n. This function is also called Tetration, Power Tower or Hyperexponentiation.

Sequence T_n converges to a finite value if $G \in [1/e^e, e^{1/e}]$. Thus, if it is defined as $T : [1/e^e, e^{1/e}] \to [1/e, e]$ the function that makes each G correspond the limit of the sequence T_n, $T(G) = z_0$ for $G \in [1/e^e, e^{1/e}]$, where z_0 is the root of f_G. Function $T(G)$ is known as the Infinite Power Tower.

Hoorfar and Hassani gave in [28, Theorem 3.1], the following bounds for the Infinite Power Tower

$$LB_T(G) = \frac{1 + \ln(1 - \ln G)}{1 - 2\ln G} \le T(G) \le \frac{\lambda + \ln(1 - \ln G)}{1 - 2\ln G} = UB_T(G) \tag{11}$$

for $G \in [1/e^e, e^{1/e}]$ where $\lambda = e - 1 - \ln(e - 1)$. Its is obtained that $LB_T(1) = T(1) = 1$ and $UB_T(e^{1/e}) = T(e^{1/e}) = e$.

To calculate the roots of the function f_G, the Newton—Raphson (N–R) method with initial seed $\mathrm{LB}_T(G)$ can be used, for values of $G > 1/e^e$. This seed, which is a lower bound, guarantees indeed the convergence of the method.

B. Calculation of the Roots of $\boldsymbol{h_x}$

To calculate the roots of the function h_x, the function $BC_4(x)$ given in (12) will be used as a seed, consisting of the first four terms of the De Bruijn-Corless-Comtet series expansion (9)

$$BC_4(x) = \ln x - \ln(\ln x) + \frac{\ln(\ln x)}{\ln x} + \frac{\ln(\ln x)(-2 + \ln(\ln x))}{2(\ln x)^2}. \tag{12}$$

One advantage of this seed is that for x values that exceed the maximum number allowed by MATLAB (Largest finite floating-point number in IEEE single or double precision: $(2 - 2^\wedge(-52)) \cdot 2^\wedge 1023 \approx 1.79769 \cdot 10^{308}$), the previously commented trick proposed in [26] when x is of the form $x = a_0 e^{b_0}$ can be used, which is precisely what happens with (2) and (3).

IV. New Numerical Methods and Explicit Formula Proposed

A. New Numerical Methods

The algorithms proposed to calculate the solutions of (1) are specified in the flowcharts given in Figs. 5 and 6.

TABLE I
SDM PARAMETERS AND ARGUMENTS FOR THE LAMBERT W FUNCTION—SCENARIOS 1–3

Parameters	I-V curve 1	I-V curve 2	I-V curve 3
I_{ph} (A)	15.88	1.032	3.654
I_{sat} (A)	$7.440 \cdot 10^{-10}$	$2.513 \cdot 10^{-6}$	$3.999 \cdot 10^{-21}$
R_s (Ω)	2.04	1.239	2.69
R_{sh} (Ω)	425.2	744.714	2329
a (V)	14.67	1.3	0.516
I_{sc} (A)	15.804	1.031	3.650
V_{oc} (V)	$3.481 \cdot 10^{2}$	16.775	24.893
x_{min} (I-approach)	$9.272 \cdot 10^{-10}$	$6.388 \cdot 10^{-6}$	$3.852 \cdot 10^{-12}$
x_{max} (I-approach)	16.758	2.513	$3.32 \cdot 10^{9}$
x_{min} (V-approach)	$1.942 \cdot 10^{-7}$	$3.86 \cdot 10^{-3}$	$3.34 \cdot 10^{-9}$
x_{max} (V-approach)	$1.686 \cdot 10^{192}$	$1.006 \cdot 10^{254}$	$9.92 \cdot 10^{7148}$

TABLE II
SDM PARAMETERS AND ARGUMENTS FOR THE LAMBERT W FUNCTION - SCENARIOS 4–6

Parameters	I-V curve 4	I-V curve 5	I-V curve 6
I_{ph} (A)	0.578	0.761	4.802
I_{sat} (A)	$1.34 \cdot 10^{-10}$	$3.107 \cdot 10^{-7}$	$4.016 \cdot 10^{-7}$
R_s (Ω)	$1.27 \cdot 10^{-2}$	0.037	$5.906 \cdot 10^{-1}$
R_{sh} (Ω)	$6.12 \cdot 10^{2}$	52.89	$1.167 \cdot 10^{3}$
a (V)	$1.18 \cdot 10^{-2}$	0.039	0.037
I_{sc} (A)	0.578	0.760	1.006
V_{oc} (V)	0.262	0.573	0.603
x_{min} (I-approach)	$2.686 \cdot 10^{-10}$	$5.939 \cdot 10^{-7}$	$1.233 \cdot 10^{28}$
x_{max} (I-approach)	1.157	1.419	$1.462 \cdot 10^{35}$
x_{min} (V-approach)	$1.649 \cdot 10^{-6}$	$8.608 \cdot 10^{-4}$	$8.33 \cdot 10^{5.2 \cdot 10^{4}}$
x_{max} (V-approach)	$12.97 \cdot 10^{2165}$	$1.028 \cdot 10^{445}$	$3.66 \cdot 10^{6.57 \cdot 10^{4}}$

These algorithms propose to use the function f_G when $x \leq e$ and the function h_x in other case. The selection of $x = e$ as the cut-off point is motivated by two facts, the first one is that the lower bound, in fact approximation, used as a seed to calculate the roots of f_G is valid, according to [28] for $x \leq e$ and, the second one, is that the series expansion (9) used to provide a seed for the roots of h_x is more accurate the larger the values of x, at least for $x > e$. Although it is possible to choose another cut-off point, it has been verified that $x = e$ provides an optimal balance in the computational performance of the two functions involved.

B. Explicit Formula

The algorithms proposed in Figs. 5 and 6 converge to a such an accurate solution in so few iterations that it is possible to consider an explicit formula just taking the first iteration for each function used. Obviously, the precision of this formula is limited but sufficient for experiments that do not require high precision but very fast output. The time taken to calculate the solution is reduced around one order of magnitude with respect to the numerical method. In next section it will be verified that the proposed formula improves, in all analyzed cases, the precision of other explicit formulas proposed in the literature, and there are no significant differences in execution time.

V. EXPERIMENTAL RESULTS

To check the validity and precision of the proposed methodologies/algorithms as well as to compare them with other existing proposals, six scenarios have been considered. Each scenario corresponds to an I-V curve with its corresponding parameters given in Tables I and II, that contemplate a wide range of possibilities. The first scenario corresponds to the *I–V* curve used in [26].

The values $x_{\min}$ and $x_{\max}$ in the tables represent the minimum and the maximum values, respectively, of the arguments of the Lambert W function in each approach, that is, the argument of the Lambert function in (2) for the I-approach, and the corresponding argument in (3) for the V-approach. The values that exceed the maximum admitted by MATLAB have been computed with DERIVE 6 program.

The results have been compared with the ones obtained by the following methods.

1) The internal lambertw function of Matlab applied to (2) and (3).
2) A sequential strategy based on the N–R method applied to (1).
3) The Hybrid Formula (13) proposed in [26] applied to (2) and (3).

Below are given some remarks about the previous methods.

i) To evaluate the precision of the methods, the results obtained with (2) and (3) after applying the lambertw function programmed in MATLAB to compute the Lambert W function images is considered as a benchmark. This methodology is referred as MATLAB lambertw.
ii) The convergence speed of the N–R method strongly depends on the proximity of the seed to the solution. Given an arbitrary fixed value of voltage V_0 or of current I_0, it is difficult to give a good seed for the N–R method applied to (1), nevertheless, if it is needed solve the equation for several values of voltages or currents in a certain interval, the values can be ordered and then use each solution as the seed of the preceding one (sequential strategy), in this way, just the first equation (the corresponding to the first voltage or current value) is initialized with a particular seed, for instance zero. As it can be seen in Fig. 7, the more points, the closer the points and the faster the average calculation time per point, but this is a trick that cannot always be applied and also gives an advantageous result of

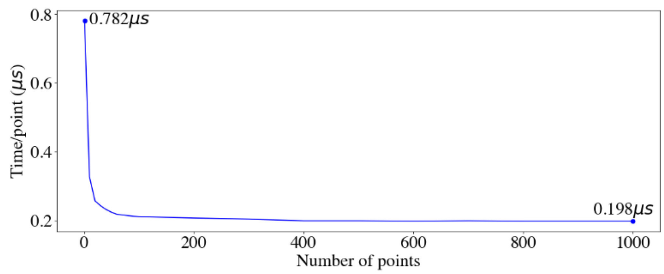

Fig. 7. Dependence of the Sequential N–R method on the number of points.

the N–R method that would not express the real mean time per point. However, this strategy has been used to analyze the speed of the other methods that are at a disadvantage. This methodology will be referred as sequential N–R method.

iii) The Hybrid Formula proposed in [26] will be used to compare the explicit method proposed here. In [26] it is asserted that it is the best option compared to other approaches, in this way, in this article, an implicitly comparison with those methods already analyzed in [26] is performed. The Hybrid Formula which consists of taking the first 6 summands of the series expansion proposed in [29] for values of the argument x of the Lambert W function in the interval $[0, 9[$, while for $x \geq 9$ it takes the first 7 summands of the series expansion (9) of De Bruijn–Corless–Comtet. Specifically

$$\mathrm{HF}(x) = \begin{cases} \mathrm{SE}_{L9}(x) & if\ 0 \leq x < 9 \\ \mathrm{SE}_{G9}(x) & if\ x \geq 9 \end{cases} \tag{13}$$

where

$$\mathrm{SE}_{L9}(x) = u + \frac{u}{1+u}p + \frac{1}{2}\frac{u}{(1+u)^3}p^2 - \frac{1}{6}\frac{u(2u-1)}{(1+u)^5}p^3 + \frac{1}{24}\frac{u(6u^2-8u+1)}{(1+u)^7}p^4 - \frac{1}{120}\frac{u(24u^3-58u^2+22u-1)}{(1+u)^9}p^5$$

with $u = \frac{x}{e}$ and $p = 1 - \frac{x}{e}$, and

$$\mathrm{SE}_{G9}(x) = L1 - L2 + \frac{L2}{L1} + \frac{L2(-2+L2)}{2(L1)^2} + \frac{L2\left(6 - 9L2 + 2(L2)^2\right)}{6(L1)^3} + \frac{L2\left(-12 + 36L2 - 22(L2)^2 + 3(L2)^3\right)}{12(L1)^4} + \frac{L2\left(60 - 300L2 + 350(L2)^2 - 125(L2)^3 + 12(L2)^4\right)}{60(L1)^5}$$

with $L1 = \ln x$ and $L2 = \ln(\ln x)$.

The errors of the methods will be computed with the root-mean-square error (RMSE) given by

$$\mathrm{RMSE}_I = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(I(V) - I_{\mathrm{aprox}}(V)\right)^2}$$

$$\mathrm{RMSE}_V = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(V(I) - V_{\mathrm{aprox}}(I)\right)^2}$$

where $I(V)$ and $V(I)$ are the "exact values" (benchmark obtained with Matlab) of the current and the voltage for data V and I, respectively, $I_{\mathrm{aprox}}(V)$ and $V_{\mathrm{aprox}}(I)$ are the corresponding approximated values obtained with the methods subject to study, and N is the number of samples. In all the study cases, we will consider $N = 1000$ points uniformly distributed between $[0, V_{\mathrm{oc}}]$ and $[0, I_{\mathrm{sc}}]$, for the *I*-approach and the *V*-approach, respectively.

Following the flowcharts provided in Section IV, the proposed numerical methods perform the necessary iterations until the relative error, given by error $\mathrm{ABS}(\frac{z_i - z_{i-1}}{z_i})$ (for function f_G) or $\mathrm{ABS}(\frac{x_i - x_{i-1}}{x_i})$ (for function h_x), is less than or equal to the tolerance fixed at 10^{-8}. Although smaller tolerances would provide more accurate results, it is enough to demonstrate the power of the proposed methodology.

A desktop computer (Intel i7-7700 3.6 GHz, 32 GB RAM, SSD disk) has been used for getting experimental results and computation times. The computation time showed in the following tables is the mean value after computing 20000 times the whole I-V curve, measured in seconds. This large number of executions guarantees stability in the measurement of times. In bold type the best results are indicated.

A. I-*Approach: Extraction of the Current in Terms of the Voltage*

The results of the *I*-approach can be seen in Table III.

The best results in precision are obtained with the new proposed numerical method, but the interesting issue is to observe that it attains accuracies of order 10^{-16} or even smaller and it is more than 1000 times faster than Matlab with its programmed lambertw function. The times reached by the new numerical method are also slightly better than those of the sequential N–R, despite the fact that the sequential N–R method is even working on an advantageous strategy that does it much faster, this also shows how powerful in speed the new numerical method is. It should also be noticed that the order of the RMSE in the sequential N–R never equals that of the new proposed numerical method, even with the minimum tolerance allowed by MATLAB.

With respect to the explicit formulas, it is also evident that they are faster but also less precise than the numerical methods although, what is interesting to notice is that, in all the cases the new explicit formula exhibits a lower error than the Hybrid Formula with hardly any differences in time.

It should be emphasized that the new numerical method is so precise that with a tolerance equal to 10^{-1} achieves a root mean square error of order less than 10^{-16} in scenario 6. Also,

TABLE III
EXPERIMENTAL RESULTS WITH THE *I*-APPROACH

	Method	RMSE	Time (s)
I-V curve 1	MATLAB lambertw	--------	0.247303
	Sequential N-R	4.284466e-15	0.000225
	New Numeric	**7.099874e-16**	**0.000215**
	Hybrid Formula	7.055238e-04	**0.000045**
	New Explicit	**4.880098e-04**	0.000047
I-V curve 2	MATLAB lambertw	--------	0.275453
	Sequential N-R	2.831386e-16	0.000239
	New Numeric	**3.040471e-17**	**0.000254**
	Hybrid Formula	1.144460e-05	0.000064
	New Explicit	**1.053385e-05**	**0.000048**
I-V curve 3	MATLAB lambertw	--------	0.246842
	Sequential N-R	1.299728e-15	0.000268
	New Numeric	**3.015232e-16**	**0.000210**
	Hybrid Formula	1.882714e-05	**0.000050**
	New Explicit	**9.103947e-06**	0.000066
I-V curve 4	MATLAB lambertw	--------	0.235025
	Sequential N-R	2.288849e-16	0.000215
	New Numeric	**1.832709e-17**	**0.000225**
	Hybrid Formula	7.723252e-06	0.000057
	New Explicit	**1.928659e-07**	**0.000027**
I-V curve 5	MATLAB lambertw	--------	0.248709
	Sequential N-R	1.845380e-16	0.000233
	New Numeric	**2.482534e-17**	**0.000231**
	Hybrid Formula	1.089631e-05	0.000057
	New Explicit	**7.729855e-07**	**0.000026**
I-V curve 6	MATLAB lambertw	--------	0.263976
	Sequential N-R	6.830052e-16	0.000283
	New Numeric	**4.294138e-16**	**0.000173**
	Hybrid Formula	3.985595e-11	0.000071
	New Explicit	**7.360368e-16**	**0.000070**

it is interesting to note (see Fig. 8) that function f_G has, in general, more leadership than h_x due to the Lambert W function argument behavior in this *I*-approach setting.

Specific information about the iterations of the functions f_G and h_x in each scenario have been also deeply analyzed and it has been observed that, for values of $x \le e$, where the root of f_G is computed, at most three iterations are performed in all the cases. Furthermore, in most approximations, only one iteration is needed to attain the tolerance. However, for values of $x > e$, where the root of h_x is computed, up to four iterations are sometimes required. Fig. 8 shows the graphs of the *I*–*V* curves in each scenario distinguishing by different type of lines the range of points where each function, f_G in solid blue line, or h_x in blue dashed line, has been applied. It is also provided the percentage of times that each function, f_G and h_x, acts on the total points that could help to understand the behavior of the proposed algorithms.

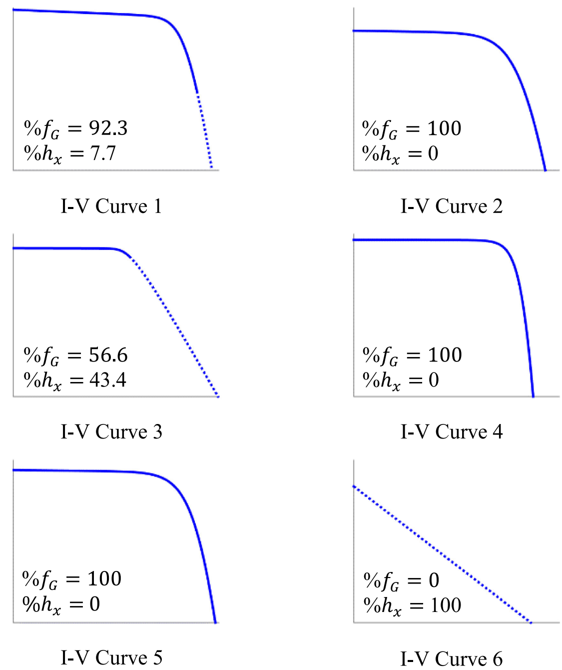

Fig. 8. *I*–*V* curves with the I-approach and actuation percentage of f_G and h_x.

TABLE IV
EXPERIMENTAL RESULTS WITH THE V-APPROACH

	Method	RMSE	Time (s)
I-V curve 1	MATLAB lambertw	---------	0.235151
	Sequential N-R	6.217746e-13	0.000277
	New Numeric	**5.078523e-13**	**0.000120**
	Hybrid Formula	4.757257e-04	**0.000048**
	New Explicit	**1.648190e-04**	0.000066
I-V curve 2	MATLAB lambertw	---------	0.232493
	Sequential N-R	6.343225e-14	0.000292
	New Numeric	**6.102022e-14**	**0.000129**
	Hybrid Formula	3.595138e-05	**0.000049**
	New Explicit	**1.340901e-05**	0.000065

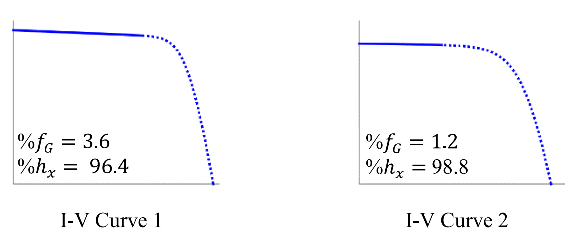

Fig. 9. *I*–*V* curves with the *V*-approach and actuation percentage of f_G and h_x.

B. V-Approach: Extraction of the Voltage in Terms of the Current

Before showing the results of the *V*-approach in Table IV, it is worth noting that only in the first two scenarios it has been possible to compare the results with the benchmark obtained by the lamberw function of MATLAB because most of the arguments of the Lambert W function exceeded the largest positive floating-point number supported by MATLAB. This agrees with that commented at the end of Section II about the usual behavior of the argument of the Lambert W function in the *V*-approach. In fact, as it is expected, in this *V*-approach, the function which predominates in the numerical algorithm is h_x (see Fig. 9).

The results in this *V*-approach are like those of the *I*-approach. The new numerical method is the most accurate and, in time, it is slightly faster than the sequential N–R method, with only one order of magnitude less quick than the explicit methods. The proposed explicit formula is again more accurate than the Hybrid Formula with hardly any differences in time.

An analysis on the number of iterations of the new numerical method has been performed. At most three iterations are necessary for both functions in all the scenarios but, furthermore, in most cases only one iteration is sufficient to attain the desired tolerance. Even in all the cases, with a tolerance of 10^{-6}, the RMSE is of the same order of the one obtained with tolerance 10^{-8}. In Fig. 9 can be seen the graphs of the *I–V* curves obtained with the *V*-approach in each scenario distinguishing by different type of lines the range of points where each function, f_G in solid blue line or h_x in blue dashed line, has been applied.

VI. Conclusion

In this article, it is proposed a new simple, fast and accurate methodology to calculate the solutions of the SDM equation. The proposed methodology allows to achieve a very high accuracy in a very short time, about three orders of magnitude faster than MATLAB with its programmed lambertw function, and it is only one order of magnitude below than some good explicit formulas, without forgetting that explicit formulas attain a much lower and fixed precision. Even, the new numerical method proposed is slightly faster than the sequential N–R method that uses a special trick to be faster when one uses a lot of points for the computation of the mean time. The method proposed achieves a very accurate solution in so few iterations that only with the first iteration it already improves in precision and with similar execution times other explicit formulas existing in the literature, so it is proposed its use as an explicit formula (taking just the first iteration) whenever very fast outputs are necessary but no a very high precision. Considering the results, the *I*-approach is computationally more appropriate than the *V*-approach because it avoids, among other things, the problems of exceeding the limits that some programs, such as MATLAB, have with excessively large numbers.

The simplicity of the proposed algorithms makes them extremely easy to program and, therefore, a good option to be used by researchers in PV solar panels modeling, specially to calculate the solutions of the SDM equation in an optimization procedure where a huge amount of such calculations are executed.

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