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In-depth analysis of single-diode model parameters from manufacturer's datasheet

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ABSTRACT

The objective of this paper is to determine all the possible combinations of the five parameters of the single-diode model (SDM) of a photovoltaic panel when only the following three important points (*remarkable points*) of a I–V curve, namely, short circuit, maximum power and open circuit points, are available, usually from manufacturer's datasheet. In this work, four of the five parameters of the SDM are expressed as explicit functions of the remaining one. Taking advantage of this, the monotony of these functions has been studied and the intervals where the corresponding parameters belong have been determined, that is, the domain of the parameters, in terms exclusively of the *remarkable points*. Using these functions, a unique SDM solution can be also easily determined if an extra data or equation is available. A possible application of this study is to validate if an extra equation is compatible with the set of equations obtained from the remarkable points. The results presented in this paper have been tested with a database gathering information of 8835 datasheets included in the Energy Commission's Solar Equipment Lists. Comparisons have also been made with other works which have tried to obtain the SDM parameters only with datasheet information.

1. Introduction and preliminaries

The combustion of fossil fuels for more than a century has generated emissions of gases such as carbon dioxide, carbon monoxide and other types which have contributed to generate and intensify the greenhouse effect, contributing for example to the acid rain, and to the air, soil and water pollution (see, for instance Ref. [1], and references therein). It is evident, in the face of the numerous meteorological disasters that are more frequent each time, that a change in the today's society energy source is vital. Renewable energies must be the solution to the energy need for industries, cities and society in general. One of the most democratized inexhaustible energy sources is solar energy. The use of solar energy depends on a suitable technology that allows to obtain a maximum electric power with the highest possible efficiency, for this purpose, photovoltaic panels are a great tool.

It is well known that the photovoltaic panels behavior can be emulated by mathematical models (equations) obtained from equivalent electrical circuits. One of the most used is the single-diode model (SDM), or 5-parameter model, which combines excellent accuracy with reasonable complexity compared to other models with more parameters (two or three-diode models).

For a solar panel with n_S cells in series and n_p cells in parallel, the SDM equation is given by

$$I = I_{ph} - I_{sat}\left(\exp\left(\frac{V + I\,R_s}{n_s n V_T}\right) - 1\right) - \frac{V + IR_s}{R_{sh}} \tag{1}$$

where I is the panel current measured in Amperes, V is the panel voltage measured in Volts, $I_{ph} = n_p I_{ph}^{cell}$ is the panel photocurrent in Amperes (the superscript "*cell*" means that the parameter corresponds to a single cell), $I_{sat} = n_p I_{sat}^{cell}$ is the panel diode saturation current in Amperes, $R_s = \frac{n_s}{n_p} R_s^{cell}$ is the panel series resistance in Ohms and, $R_{sh} = \frac{n_s}{n_p} R_{sh}^{cell}$ is the panel shunt resistance in Ohms. The value n is the ideality factor, and $V_T = \frac{k}{q} T$ is the so-called thermal

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voltage, where T is the temperature in Kelvin degrees, k is the Boltzmann's constant and, q the electron charge.

Equation (1) can be rewritten in the following alternative form (see Ref. [2])

$$I = A + B - EV - BC^V D^I \tag{2}$$

where

$$A = \frac{I_{ph}\, R_{sh}}{R_{sh} + R_s}\ ,\ B = \frac{I_{sat}\, R_{sh}}{R_{sh} + R_s}\ ,\ C = e^{\frac{1}{n_s n V_T}}\ ,\ D = e^{\frac{R_s}{n_s n V_T}}\ ,\ E = \frac{1}{R_{sh} + R_s} \tag{3}$$

The main advantage of equation (2) is that using the geometrical information of the new parameters, it is possible to perform methods for extracting such parameters from I–V data real measurements (see Refs. [2,3]).

There are many methods in the literature to extract the SDM parameters from the I–V data obtained from photovoltaic panels real measurements. Essentially, it is possible to distinguish between the methods that try to fit a real I–V curve by some optimization procedure, for example, minimizing some difference/error between the real data and the theoretical I–V curve (see, for example, [4–6]), and those that approximate the SDM neglecting some term declared not significant or approximating some term by a quantity estimated empirically, and then obtaining an explicit solution of the approximate model (see, for example, [7–9]). There exists a particular method [10] that, using only the coordinates of four arbitrary points of the I–V curve with the corresponding slopes, is able to reduce the exact SDM determination to compute a five-degree polynomial equation. Other methods try to determine the SDM using only the data provided in the solar panel manufacturers datasheets, that is the subject of the present paper. Usually, the data used in these methods are the so-called *remarkable points*: the open circuit point, the maximum power point ($MPP = (V_{MPP}, I_{MPP})$) and the short circuit point, normally in standard conditions (STC). The variation ratios of some of these points with respect to a change in temperature, called temperature coefficients, are also eventually used. It should be added that, taking into account the maximum power condition at the MPP, the I–V curve slope at this point, say I'_{MPP}, is also known, concretely, $I'_{MPP} = -I_{MPP}/V_{MPP}$. From now on, it will be said that a I–V curve satisfies the *remarkable points conditions* if it crosses the remarkable points and it has slope $-I_{MPP}/V_{MPP}$ at the MPP: these are consequently double conditions for the I–V curve.

Taking into account that the SDM has five parameters to be determined, and only four conditions to be satisfied, there are infinite SDM I–V curves satisfying the remarkable points conditions (see Ref. [11]). Therefore, in order to uniquely determine the SDM, it is necessary to use an extra datum, for example, a point of the I–V curve different from the remarkable ones, the slope at the short-circuit or at the open-circuit point, the value of a parameter, or an extra equation that relates the five parameters to each other, which would mean that they are not independent. We would like to emphasize that only using the remarkable points of a datasheet, without an extra datum, it makes no sense to say that there is a best solution for the SDM satisfying the remarkable points conditions because there are infinite exact SDM solutions.

Many papers have tried to provide a unique SDM solution using an extra datum apart of the remarkable points provided in the datasheet. Two issues appear, the first one is to select and justify the extra datum, which should lead, at least, to a feasible solution between the infinite possible ones; the second one is to compute, taking into account the extra datum, the corresponding SDM parameters, that is, to determine the model with a proposed methodology.

Some papers set a fixed value for one parameter as extra datum and try to find the unique SDM solution. For example, in Villalba et al. [12] it is assumed that the ideality factor is constant and can be arbitrarily chosen, usually between 1 and 1.5. As it is demonstrated in the present paper (and also in Ref. [11]), there exist infinite possible values of the ideality factor which provide possible solutions satisfying the remarkable points conditions, but the interval $[1, 1.5]$ declared in Ref. [12] is far from being valid generically. Villalba et al. take the ideality factor equal to 1.3 and, then, describe a methodology to obtain a solution of the SDM which satisfy approximately the remarkable points conditions. Their approach depends on some parameter's formulas in terms of the temperature and the irradiance which include the temperature coefficients at the short circuit and open circuit points. The accuracy of their methodology strongly depends on the parameter's temperature-irradiance formulas accuracy, for this reason, it is also mentioned there that the ideality factor can be modified a posteriori to obtain a better fit of the model. In the present paper, given a value for the ideality factor, for instance 1.3, the SDM is univocally and straightforwardly determined just by solving an equation with a unique unknown and, then, replacing the obtained solution in the parameters explicit expressions which only depend of this unknown, no more extra data is needed at all.

In Ref. [13], using the remarkable points and the temperature coefficients at short circuit and open circuit points, a formula for computing the ideality factor in standard conditions is given. In the present paper, the domain of the ideality factor will be provided and, so, it will be possible to check if the formula in [13] provides values compatible with the remarkable points conditions.

Another fifth equation very well-known in the literature is presented by De Soto et al. in Ref. [14]. To obtain it, they consider certain empirical expressions of the parameters as functions of the temperature and the irradiance involving the temperature coefficients at the open and short circuit points. When these parameter expressions are imposed on the SDM equation for a certain given increment of temperature, a new equation is obtained. The question which arises is if this new equation is compatible with the remarkable points conditions. Using the methodology of the present paper, it is possible to obtain, for each increment of temperature, the SDM solution after solving a one-variable equation.

Other papers use heuristic rules to provide the extra datum. For example, [11] proposes as a possibility to consider the value of the ideality factor equal to $0.9n_{max}$ where n_{max} is the maximum possible value of the ideality factor under the remarkable points conditions, in that paper it is provided moreover a methodology based on a Reduced Form technique [15,16] to obtain in such a case the unique SDM solution.

Also papers can be found in the literature that use certain simplifications of the SDM model equation to obtain an approximate SDM solution. The simplifications are directly done in the model equation, for example, neglecting the series resistance or the shunt resistance (see, for instance, Refs. [17,18]), or a posteriori on the equations obtained from the SDM after applying the remarkable points conditions (see Refs. [19,20]). An example of the first type can be found in Ref. [17] which assumes that the shunt resistance is so high that it can be considered infinite and, so, the SDM becomes a four parameters model. Theoretically, this hypothesis determines a unique SDM solution, nevertheless, in [17] the open circuit and the short circuit temperature coefficients are used to obtain an approximated solution. In the present paper, the solution for this case is simply and explicitly obtained and, so, only a substitution in the formulas is needed, without any more information, therefore, the use of extra data could only lead to incompatibilities. An example of the second type of simplifications can be found in Ref. [19] where, using the Lambert W function, the authors provide an explicit SDM solution depending on the ideality factor value which is the extra datum needed to attain the unique approximate solution. With respect to the second type of simplifications, it should be commented that, if a single solution were achieved without an extra datum, it could be due, most probably, to the strong restrictions coming from the simplifications.

Other possibilities to obtain a unique SDM solution are based in using a fifth complementary equation of the four ones obtained after applying the remarkable points conditions [21–23]. For example, in Ref. [21] it is supposed that the shunt resistance is opposite and inverse to the I–V curve slope at the short circuit point. In the present paper, this equation is directly transformed in an equivalent equation with only one unknown whose solution can be easily obtained with a numerical method, after that, the SDM is completely determined by substituting the obtained solution in the parameters explicit expressions. As commented before, an important question will be to check if this new equation is compatible with the remarkable points conditions. In Ref. [24], a relation between the diode ideality factor and the open circuit voltage is used as a fifth equation. The relation provided is explicitly formulated using the temperature coefficients of the short circuit and open circuit points and, moreover, certain simplifications of the equations after applying the remarkable points conditions are also done.

It is also possible to find papers (see, for instance, Refs. [25,26]) that try to solve the double-diode model (DDM) with seven parameters just by using the remarkable points conditions, obviously there are much more unknowns than data, then, it is necessary more extra information even than with the SDM. For example, in Ref. [26], the series and shunt resistances are both neglected together with a relationship between the two reverse saturation current diodes, reducing the DDM to four parameters. A particular solution is found with the methodology proposed there.

In the present work, a methodology is provided to obtain straightforwardly the infinite SDM curves satisfying the remarkable points conditions. The key idea is to express four parameters as functions of the remaining one which will play the role of independent variable and, then, to demonstrate the monotony of these four functions. Knowing the exact interval or domain of the parameter acting as the independent variable, it is provided the range of the functions, in other words, the intervals where each one of the five parameters belong. The knowledge of these intervals, which are actually the domains of the parameters satisfying the remarkable points conditions, will allow to verify if the parameters obtained in some articles are or not feasible and, if a new fifth equation is compatible with the remarkable points conditions, in other words, this could be a way to verify when a certain fifth equation is valid or not. Another application of the knowledge of these intervals could be its use in metaheuristic algorithms that need strongly to know a priori the intervals where the solution belongs to ensure the convergence of them. A large number of these metaheuristic techniques are being more and more applied to extract the SDM parameters. Some examples of these optimization methods are the following algorithms: genetic [27], particle swarm [28], pattern search [29], simulated annealing [30], artificial bee colony [31], adaptive differential evolution [32], harmony search-based [33], or salp swarm [34], among many others.

Finally, just emphasize that, if an extra datum is available, the determination of the unique SDM solution is reduced to solve a one-variable equation. This extra datum could be an extra point different of the remarkable ones, any parameter, the slope at the short circuit point, or a fifth equation relating the parameters.

2. Exact resolution of the single-diode model with datasheet points

The objective of this section is to obtain the parameters of the model equation (2) with the following data provided in the photovoltaic panels datasheets. We will refer to them as the *remarkable datasheet points*:

- The short-circuit current I_{SC}.
- The maximum power point (V_{MPP}, I_{MPP}) with its property $I'_{MPP} = -\frac{I_{MPP}}{V_{MPP}}$.
- The open-circuit voltage V_{OC}.

Remark: Related to the property at the maximum power point (MPP), just remember that it is a consequence to the fact that, at this point, the power $P = I \cdot V$ attains its maximum and, taking into account the properties of I as a function of V (see, for instance, Ref. [35]), it is satisfied that $P' = \frac{dP}{dV} = 0$ at the MPP, which implies that the slope of the I–V curve at the MPP, I'_{MPP}, must satisfy that $I'_{MPP} = \frac{dI}{dV}(V_{MPP}) = -\frac{I_{MPP}}{V_{MPP}}$.

2.1. Maximum power and open circuit conditions

Taking logarithms on both sides of (2), the following equation is obtained

$$\ln(B) + V\ln(C) + I\ln(D) = \ln(K - EV - I) \tag{4}$$

where $K = A + B$. Now, differentiating in equation (4) with respect to V (recall that I varies dependently on V [35]).

$$\ln(C) + I'\ln(D) = \frac{-E - I'}{K - EV - I} \tag{5}$$

A theoretical I–V curve obtained from model (2) crossing the maximum power and the open circuit points, with slope I'_{MPP} at the maximum power point, is obtained forcing equations (4) and (5) to satisfy these conditions, giving rise to the following system

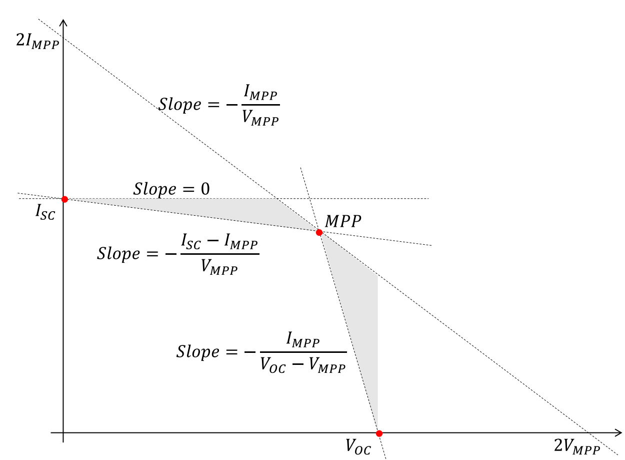

Fig. 1. I–V curves feasible region.

$$\begin{bmatrix} 1 & V_{MPP} & I_{MPP} \\ 1 & V_{OC} & 0 \\ 0 & 1 & -I_{MPP}/V_{MPP} \end{bmatrix} \begin{bmatrix} \ln B \\ \ln C \\ \ln D \end{bmatrix} = \begin{bmatrix} \ln(K - EV_{MPP} - I_{MPP}) \\ \ln(K - EV_{OC}) \\ -\dfrac{E - I_{MPP}/V_{MPP}}{K - EV_{MPP} - I_{MPP}} \end{bmatrix} \tag{6}$$

Denoting $\Omega = \begin{bmatrix} 1 & V_{MPP} & I_{MPP} \\ 1 & V_{OC} & 0 \\ 0 & 1 & -I_{MPP}/V_{MPP} \end{bmatrix}$, it is satisfied, due to the strict concavity of the I–V curve [35] and the fact that $V_{OC} < 2V_{MPP}$ (see Fig. 1), that $det\Omega = \frac{I_{MPP}}{V_{MPP}}(2V_{MPP} - V_{OC}) \neq 0$. Therefore, system (6) has always solution for those values of K and E such that the elements of the independent term of (6) exist, that is, when conditions (7) are satisfied.

$$E < \frac{K - I_{MPP}}{V_{MPP}} \text{ and } E < \frac{K}{V_{OC}} \tag{7}$$

In this case, the solution of system (6) is given by

$$\begin{bmatrix} \ln B \\ \ln C \\ \ln D \end{bmatrix} = \Omega^{-1} \begin{bmatrix} \ln(K - EV_{MPP} - I_{MPP}) \\ \ln(K - EV_{OC}) \\ -\dfrac{E - I_{MPP}/V_{MPP}}{K - EV_{MPP} - I_{MPP}} \end{bmatrix} \tag{8}$$

where

$$\Omega^{-1} = \frac{1}{2V_{MPP} - V_{OC}} \begin{bmatrix} -V_{OC} & 2V_{MPP} & -V_{MPP}V_{OC} \\ 1 & -1 & V_{MPP} \\ \dfrac{V_{MPP}}{I_{MPP}} & -\dfrac{V_{MPP}}{I_{MPP}} & \dfrac{V_{MPP}(V_{OC} - V_{MPP})}{I_{MPP}} \end{bmatrix}$$

From (8), B, C and D can be immediately obtained as functions of K and E. Then, denoting them as $B(K, E)$, $C(K, E)$, and $D(K, E)$, equation (2) becomes (9),

$$I = K - EV - B(K,E)\, C(K,E)^V D(K,E)^I \tag{9}$$

where the unique undetermined parameters are K and E.

2.2. Short circuit condition

Now, forcing the I–V curve to cross the short circuit point, the following equation, with variables K and E, coming from (9) must be satisfied:

$$I_{SC} = K - B(K,E)D(K,E)^{I_{SC}} \tag{10}$$

Observe that equation (10) allows us to define E as a function of K, or K as a function of E .

3. The domain of the single diode-model parameters

3.1. The domain of the parameter *E* and the infinite datasheet I–V curves

In [35] it was demonstrated that the I–V curve obtained from the SDM equation (2) has an oblique asymptote given by $K - EV$. This fact, together with the geometric properties of the model I–V curve, implies that the value E must necessarily be in the following interval

$$Interval(E) = \left]0, \frac{I_{SC} - I_{MPP}}{V_{MPP}}\right[\tag{11}$$

(i.e. $0 < E < \frac{I_{SC} - I_{MPP}}{V_{MPP}}$), as can be visualized in Fig. 1.

Each one of the possible values of E in the interval $\left]0, \frac{I_{SC} - I_{MPP}}{V_{MPP}}\right[$ determines a I–V curve lying exactly in the grey zone of Fig. 1.

Therefore, an infinite number of I–V curves can be obtained satisfying the short circuit, the maximum power and the open circuit conditions. See also [11] in this regard.

3.1.1. Extreme cases

The extreme case $E = 0$ can also be achieved throughout the proposed methodology. The I–V curve obtained in this extreme case corresponds to the SDM with $R_{sh} = +\infty$.

Observe that, in this extreme case, only the three points of the datasheet completely determines the SDM.

The extreme case when $E = \frac{I_{SC} - I_{MPP}}{V_{MPP}}$ does not provide a particular solution of the Single Diode Model because the corresponding function is not differentiable in its domain.

3.2. The parameter *K*

In [2] it was proposed an analytical method to extract the parameters of the SDM. This method, called Oblique Asymptote (OA) Method, is based on the geometric properties of the theoretical I–V curve obtained from the SDM. The main idea of this method is to assume that the I–V curve behaves as a line (oblique asymptote) very near to the short circuit point and this leads to consider $K = I_{SC}$. Recently, Batzelis in Ref. [36] has proven that the OA Method is one of the best analytical methods in the literature.

Also, observe that $K = I_{SC}$ or, equivalently, $A + B = I_{SC}$, can be expressed in terms of the original parameters of the model as:

$$I_{ph} = \frac{R_{sh} + R_s}{R_{sh}} I_{SC} - I_{sat} \tag{12}$$

Villalba et al. in Ref. [37] used a similar hypothesis but misprizing I_{sat}

$$I_{ph} = \frac{R_{sh} + R_s}{R_{sh}} I_{SC} \tag{13}$$

which is equivalent to $A = I_{SC}$.

The previous similarity between hypotheses (12) and (13) supports the assumption

$$K = I_{SC} \tag{14}$$

Therefore, using (13) in the original model equation (2), an approximate single-diode model equation is obtained:

$$I = I_{SC} - EV - BC^V D^I \tag{15}$$

Obviously, the parameters B, C, D, E, and $A = I_{SC} - B$ of this approximate model, as well as the corresponding I_{ph}, I_{sat}, n, R_s, and R_{sh} obtained with the relations (3), are not exactly the same as those of the corresponding original models, although they are very close.

Imposing to this new model (15) the open circuit and the maximum power conditions, and proceeding in the same way as in (2), the same system (6) and the same solution (8) are obtained but writing I_{SC} instead of K. The explicit expressions of parameters B, C

and D, which now only depend on E, are given by

$$B(E)=\exp\left(\frac{1}{2V_{MPP}-V_{OC}}\left(-V_{OC}\ln(I_{SC}-EV_{MPP}-I_{MPP})+2V_{MPP}\ln(I_{SC}-EV_{OC})+V_{MPP}V_{OC}\frac{E-\frac{I_{MPP}}{V_{MPP}}}{I_{SC}-EV_{MPP}-I_{MPP}}\right)\right) \tag{16}$$

$$C(E)=\exp\left(\frac{1}{2V_{MPP}-V_{OC}}\left(\ln(I_{SC}-EV_{MPP}-I_{MPP})-\ln(I_{SC}-EV_{OC})-V_{MPP}\frac{E-\frac{I_{MPP}}{V_{MPP}}}{I_{SC}-EV_{MPP}-I_{MPP}}\right)\right) \tag{17}$$

$$D(E)=\exp\left(\frac{1}{2V_{MPP}-V_{OC}}\left(\frac{V_{MPP}}{I_{MPP}}\ln(I_{SC}-EV_{MPP}-I_{MPP})-\frac{V_{MPP}}{I_{MPP}}\ln(I_{SC}-EV_{OC})-\frac{V_{MPP}(V_{OC}-V_{MPP})}{I_{MPP}}\frac{E-\frac{I_{MPP}}{V_{MPP}}}{I_{SC}-EV_{MPP}-I_{MPP}}\right)\right) \tag{18}$$

I–V curves satisfying the equation (15) with $B = B(E)$, $C = C(E)$, and $D = D(E)$, satisfy exactly maximum power and open circuit conditions and almost exactly the short circuit one for each $E\in\left]0,\frac{I_{SC}-I_{MPP}}{V_{MPP}}\right[$, in fact, the error between the I_{SC} of the datasheet and the current one, I_{SCE}, obtained from equation (15) when $V = 0$, is bounded as

$$0<I_{SC}-I_{SCE}<B(E)D(E)^{I_{SC}} \tag{19}$$

It will be proved in subsection 5.3 that I_{SCE} is very close to I_{SC} for each E but, more important, this approximation will allow to obtain almost exactly the domains of the remaining parameters.

Needless to say, the possible infinite datasheet I–V curves, actually almost exact, are now straightforwardly extracted from (15) for each possible value of E. So, from now on, the model (15) will be used.

3.3. The ideality factor *n* through the parameter *C*

In order to determine the interval where the ideality factor belongs, the parameter C has to be deeply studied.

The following simplified expression of C can be obtained after an algebraic handling of (17):

$$C(E)=\exp(k_1(k_2+g(E))) \tag{20}$$

where

$$g(E)=ln\left(\frac{\alpha-E}{\beta-E}\right)+\frac{\gamma-E}{\alpha-E} \tag{21}$$

and

$$k_1=\frac{1}{2V_{MPP}-V_{OC}},\ k_2=\ln\left(\frac{V_{MPP}}{V_{OC}}\right)\ \alpha=\frac{I_{SC}-I_{MPP}}{V_{MPP}},\ \beta=\frac{I_{SC}}{V_{OC}}\ \text{and}\ \gamma=\frac{I_{MPP}}{V_{MPP}} \tag{22}$$

The geometrical meaning of constants α, β and γ can be visualized in Fig. 2.

It can be easily checked (see Fig. 2) that

$$0<\alpha<\beta<\alpha+\gamma<2\gamma \tag{23}$$

Remark: For the data provided in the 8835 modules included in the Energy Commission's Solar Equipment Lists [38] (actually, in the CEC database there are more entries but many of them are repetitions or incorrect), it has been checked that the inequality $\frac{2}{3}I_{SC}\le I_{MPP}$ is satisfied, or equivalently,

$$3I_{MPP}-2I_{SC}\ge 0 \tag{24}$$

Observe that (24) can be expressed in terms of the constants (22) as

$$\gamma-2\alpha\ge 0 \tag{25}$$

As can be seen in Table 1, effectively the hypothesis (25) is satisfied in all the cases and for all technologies analyzed without exception.

3.3.1. Monotonicity of function *C* in $]0,\alpha[$

From $g(E)$ in equation (21), $g'(E)=\frac{\alpha^2-2\alpha\beta+\beta\gamma+E(\beta-\gamma)}{(\alpha-E)^2(\beta-E)}$ is obtained, note that g' is a continuous function in $]-\infty,\alpha[$ and, if $\gamma\neq\beta$, $g'(E)=0$ if, and only if, $E=\overline{E}$ where

$$\overline{E}=\frac{\alpha^2-2\alpha\beta+\beta\gamma}{\gamma-\beta}=\alpha-\frac{(\beta-\alpha)(\gamma-\alpha)}{\beta-\gamma}=\beta+\frac{(\alpha-\beta)^2}{\gamma-\beta} \tag{26}$$

Observe that.

- If $\gamma=\beta$ then $g'(E)=\frac{(\alpha-\beta)^2}{(\beta-E)(\alpha-E)^2}>0$ for all $E\in]0,\alpha[$.
- If $\gamma>\beta$ then $\overline{E}>\beta>\alpha$, so, $\overline{E}\notin]0,\alpha[$, therefore, g' has constant sign in $]0,\alpha[$ and, since $\lim_{E\to\alpha^-}g'(E)=+\infty$, then $g'(E)>0$ for all $E\in]0,\alpha[$.

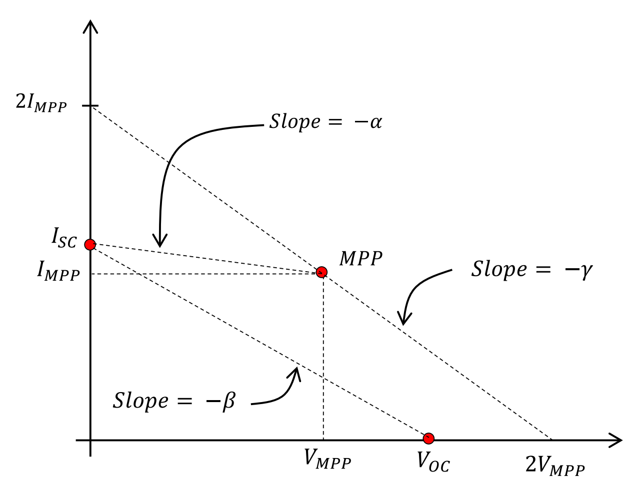

Fig. 2. Slopes α, β and γ.

Table 1
$\gamma - 2\alpha$ check in CEC PV Database.

$\gamma - 2\alpha$				
Technology	Number of Modules	Maximum	Minimum	Average
Monocrystalline	3613	2.545	0.047	0.193
Polycrystalline	4601	1.198	0.065	0.222
Thin Film	621	1.714	0.002	0.083

- If $\gamma < \beta$, taking into account (23) and (25), $\overline{E} = \frac{\alpha^2 - 2\alpha\beta + \beta\gamma}{\gamma - \beta} = \frac{(\beta-\alpha)(\gamma-2\alpha)+\alpha(\gamma-\alpha)}{\gamma-\beta} \leq 0$. Since $g'(E) = 0$ only at the point $E = \overline{E}$, and $\lim_{E\to\alpha^-} g'(E) = +\infty$, then $g'(E) > 0$ for $E > \overline{E}$, in particular, $g'(E) > 0$ for all $E \in]0, \alpha[$.

From the previous properties it can be concluded that, under (24), $g'(E) > 0$ for all $E \in]0, \alpha[$, so $C(E)$ is increasing in $]0, \alpha[$.

3.3.2. *The ideality factor* $\boldsymbol{n}$

The ideality factor can be written as a function of E like

$$n(E) = \frac{1}{n_s V_T \ln(C(E))} \tag{27}$$

The previous properties of C lead to the following proposition.

Proposition: Under (14) and (24), the ideality factor n is a decreasing function of E in $]0, \alpha[$ and, consequently,

$$n \in]n(\alpha), n(0)[\tag{28}$$

where, since $\lim_{E\to\alpha^-} C(E) = +\infty$,

$$n(\alpha) = \lim_{E\to\alpha^-} n(E) = 0 \tag{29}$$

where $\lim_{E\to\alpha^-}$ means "limit when E tends to α by the left", and, by other hand,

$$n(0) = \frac{1}{n_s V_T \ln(C(0))} = \frac{q(2V_{MPP} - V_{OC})}{n_s\, k\, T \left(\ln\left(\frac{I_{SC}-I_{MPP}}{I_{SC}}\right) + \frac{I_{MPP}}{I_{SC}-I_{MPP}}\right)} \tag{30}$$

Fig. 3 illustrates the ideality factor behaviour.

3.4. *The series resistance* $\boldsymbol{R_s}$

The following simplified expression of parameter D can be obtained after an algebraic handling of (18).

$$D(E) = \exp\left(\frac{k_1}{\gamma}\left(k_2 + g(E) + \left(\frac{\alpha+\gamma}{\beta} - 2\right)\frac{\gamma - E}{\alpha - E}\right)\right) \tag{31}$$

where g was defined in (21) and k_1, k_2, α, β, and γ were defined in (22).

The series resistance can be written as a function of E like

$$R_s(E) = \frac{\ln(D(E))}{\ln(C(E))} = \frac{1}{\gamma}\left(1 + \left(\frac{\alpha+\gamma}{\beta} - 2\right)\frac{1}{k_2 + g(E)}\frac{\gamma - E}{\alpha - E}\right) \tag{32}$$

The derivative of R_s with respect to E is given by

$$R_s'(E) = \left(2 - \frac{\alpha+\gamma}{\beta}\right)\frac{\gamma - \alpha}{\gamma(k_2 + g(E))^2(\alpha - E)^2} h(E) \tag{33}$$

where

$$\begin{aligned} h(E) &= \frac{1}{\gamma - \alpha} g'(E)(\alpha - E)(\gamma - E) - (k_2 + g(E)) \\ &= \frac{\alpha - \beta}{\gamma - \alpha}\frac{\gamma - E}{\beta - E} + \ln\left(\frac{V_{OC}}{V_{MPP}}\right) + \ln\left(\frac{\beta - E}{\alpha - E}\right) \end{aligned} \tag{34}$$

with $0 < 2 - \frac{\alpha+\gamma}{\beta} < 1$ and $\frac{\gamma-\alpha}{\gamma(k_2+g(E))^2(\alpha-E)^2} > 0$.

Therefore, the sign of R_s' coincides with the sign of h.

The derivative of h is $h'(E) = \frac{(\beta-\alpha)^2(\gamma-E)}{(\alpha-E)(\beta-E)^2(\gamma-\alpha)} > 0$ for $E < \alpha$, then h is increasing in $]0, \alpha[$.Since $\lim_{E\to\alpha^-} h(E) = +\infty$, two complementary cases in terms of the sign of $h(0)$ can be distinguished, where

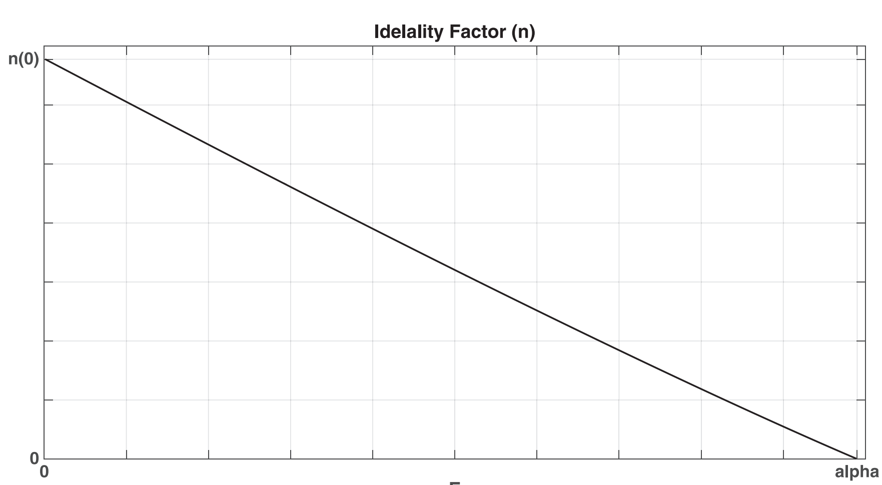

Fig. 3. The ideality factor n as a function of E.

Table 2
h(0) values in CEC PV database.

$h(0)$				
Technology	Number of Modules	Maximum	Minimum	Average
Monocrystalline	3613	3.422	0.877	1.733
Polycrystalline	4601	4.618	0.780	1.717
Thin Film	621	1.994	0.186	1.083

$$h(0) = ln\left(\frac{I_{SC}}{I_{SC} - I_{MPP}}\right) - \frac{1}{2}\frac{I_{MPP}}{V_{MPP}}\left(\frac{2V_{MPP} - V_{OC}}{2I_{MPP} - I_{SC}} + \frac{V_{OC}}{I_{SC}}\right) \tag{35}$$

i If $h(0) \geq 0$ then $h > 0$ in $]0, \alpha[$ and, consequently, $R_s' > 0$ in $]0, \alpha[$.
ii If $h(0) < 0$ then there exists a unique $\tilde{E} \in]0, \alpha[$ such that $h(\tilde{E}) = 0$, which implies that $h < 0$ in $]0, \tilde{E}[$ and $h > 0$ in $]\tilde{E}, \alpha[$. Consequently, $R_s' < 0$ in $]0, \tilde{E}[$ and $R_s' > 0$ in $]\tilde{E}, \alpha[$.

Remark: As can be seen in Table 2, it has been verified that $h(0) \geq 0$ for all datasheets provided in the CEC database [38], so, it could be used as a common assumption and it will be named Hypothesis (36).

$$h(0) \geq 0 \tag{36}$$

As a consequence of the previous properties, the following result can be established.

Proposition: Under (14) and (36), the series resistance R_s is an increasing function of E in $]0, \alpha[$ and, as a consequence,

$$R_s \in]R_s(0), R_s(\alpha)[\tag{37}$$

where

$$R_s(\alpha) = \lim_{E \to \alpha^-} R_s(E) = \frac{1}{\gamma}\left(\frac{\alpha + \gamma}{\beta} - 1\right) = \frac{V_{OC} - V_{MPP}}{I_{MPP}} \tag{38}$$

and

$$R_s(0) = \lim_{E \to 0^+} R_s(E) = \frac{1}{\gamma}\left(1 + \left(\frac{\alpha + \gamma}{\beta} - 2\right)\frac{\frac{\gamma}{\alpha}}{k_2 + \ln\left(\frac{\alpha}{\beta}\right) + \frac{\gamma}{\alpha}}\right)$$
$$= \frac{V_{MPP}}{I_{MPP}}\left(1 + \left(\frac{V_{OC}}{V_{MPP}} - 2\right)\left(\frac{\frac{I_{MPP}}{I_{SC} - I_{MPP}}}{\ln\left(\frac{I_{SC} - I_{MPP}}{I_{SC}}\right) + \frac{I_{MPP}}{I_{SC} - I_{MPP}}}\right)\right) \tag{39}$$

Fig. 4 illustrates the behaviour of R_s.

In equations (19) and (20) of [15], using the Lambert W function, an upper bound for R_s was provided as a function of the ideality factor n, assuming that $0.5 \leq n \leq 2$, only depending on the remarkable datasheet points.

3.5. *The shunt resistance* R_{sh}

Since $E = \frac{1}{R_{sh} + R_s}$ (recall (3)), the shunt resistance can be written as a function of E like

$$R_{sh}(E) = \frac{1}{E} - R_s(E) \tag{40}$$

Now, taking into account the properties of R_s obtained in the previous subsection, the following result is obtained.

Proposition: Under (14) and (36), the shunt resistance function R_{sh} is decreasing in $]0, \alpha[$ and, consequently,

$$R_{sh} \in]R_{sh}(\alpha), +\infty[\tag{41}$$

where

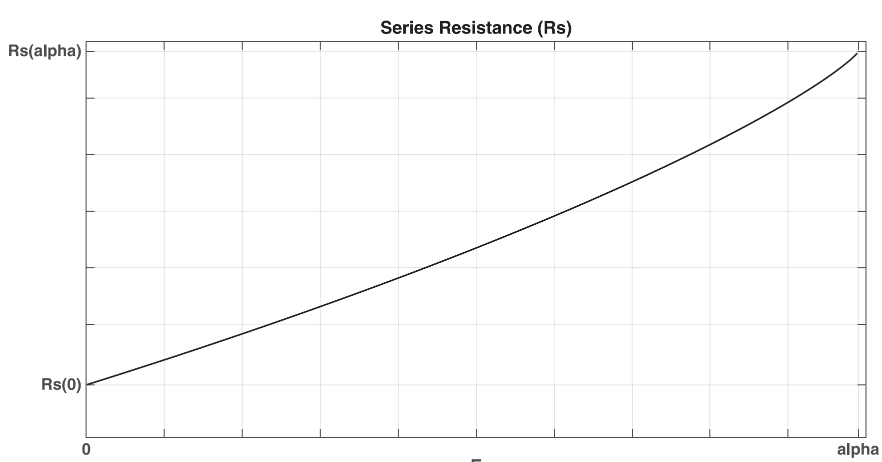

Fig. 4. The series resistance R_s in [Ω] as a function of E in [Ω^{-1}].

Fig. 5. The shunt resistance R_{sh} in [Ω] as a function of E in [Ω^{-1}].

$$R_{sh}(\alpha)=\frac{1}{\alpha}-R_s(\alpha)=\frac{V_{MPP}}{I_{SC}-I_{MPP}}-\frac{V_{OC}-V_{MPP}}{I_{MPP}}=\frac{V_{OC}(2I_{MPP}-I_{SC})+I_{SC}(2V_{MPP}-V_{OC})}{2I_{MPP}(I_{SC}-I_{MPP})}>0 \tag{42}$$

Fig. 5 illustrates the behaviour of R_{sh}.

The lower bound provided in the interval of (41) of the previous proposition was already given by Villalba et al. [37] although, in that paper, this value was simply guessed by a geometrical idea and was used as initial seed in their algorithm.

3.6. The diode reverse saturation current $\mathbf{I_{sat}}$

The diode reverse saturation current can be expressed as a function of E as

$$I_{sat}(E)=B(E)\left(1+\frac{R_s(E)}{R_{sh}(E)}\right) \tag{43}$$

where function B comes from equation (16). Regrouping conveniently, B can be expressed in terms of C as

$$B(E)=\exp(\ln(I_{SC}-EV_{OC})-V_{OC}\ln(C(E))) \tag{44}$$

Recall that, under Hypothesis (24), function C is increasing in $]0,\alpha[$, so, from (44) B is decreasing in $]0,\alpha[$ and, so,

$$B\in]B(\alpha),B(0)[=]0,B(0)[\tag{45}$$

where

$$B(0)=\exp\left(\ln(I_{SC})-V_{OC}\ln(C(0))\right)=\exp\left(\ln(I_{SC})-\frac{V_{OC}}{2V_{MPP}-V_{OC}}\left(\ln\left(\frac{I_{SC}-I_{MPP}}{I_{SC}}\right)+\frac{I_{MPP}}{I_{SC}-I_{MPP}}\right)\right) \tag{46}$$

The fact that $B(\alpha)=\lim_{E\to\alpha^-}B(E)=0$ is a consequence of $\lim_{E\to\alpha^-}C(E)=+\infty$ and $\ln(I_{SC}-\alpha V_{OC})$ is finite.

The objective along the paper has been to prove the monotony of each parameter studied and, as a consequence, to stablish the interval where the corresponding parameter belongs to. In the case of I_{sat}, it has not been possible to demonstrate theoretically that I_{sat} is a monotonic function of E in $]0,\alpha[$, nevertheless, it has been checked numerically for all datasheets in the CEC database [38]. Specifically, for each datasheet, one hundred different E values have been taken uniformly distributed in $]0,\alpha[$ and it has been verified that I_{sat} behaves in a decreasing way for this set of points (see Fig. 6), as a consequence, it is deduced that

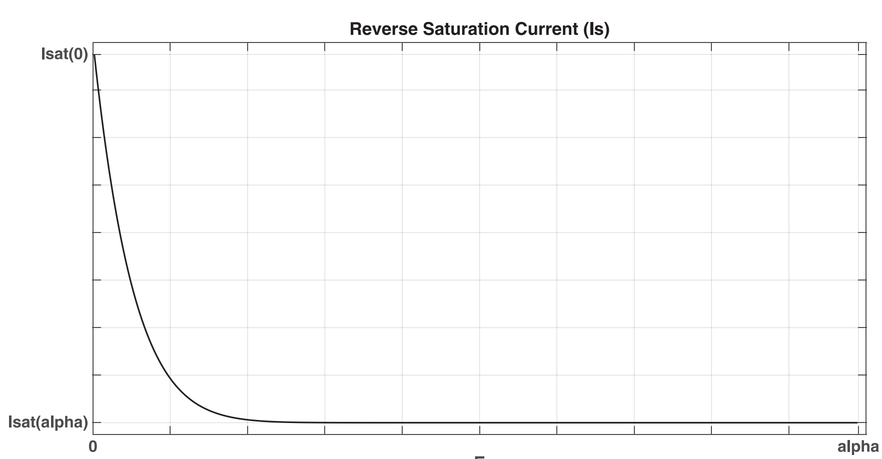

Fig. 6. The diode reverse saturation current I_{sat} in [A] as a function of E [Ω^{-1}].

$$I_{sat} \in]0\ ,\ I_{sat}(0)[\tag{47}$$

where

$$I_{sat}(0) = \exp\left(\ln(I_{SC}) - \frac{V_{OC}}{2V_{MPP} - V_{OC}}\left(\ln\left(\frac{I_{SC} - I_{MPP}}{I_{SC}}\right) + \frac{I_{MPP}}{I_{SC} - I_{MPP}}\right)\right) \tag{48}$$

3.7. The photocurrent I_{ph}

From (14) $A + B = K = I_{SC}$, then

$$A(E) = I_{sc} - B(E) \tag{49}$$

So, from the properties of $B(E)$ obtained in the previous subsection, A is increasing in $]0, \alpha[$, and

$$A \in]A(0), A(\alpha)[=]I_{sc} - B(0), I_{sc}[\tag{50}$$

The photocurrent can be expressed as a function of E as

$$I_{ph}(E) = A(E)\left(1 + \frac{R_s(E)}{R_{sh}(E)}\right) = I_{sc}\left(1 + \frac{R_s(E)}{R_{sh}(E)}\right) - I_{sat}(E) \tag{51}$$

The derivative function of I_{ph} is given by

$$I'_{ph}(E) = I_{sc}\left(\frac{R'_s(E)R_{sh}(E) - R_s(E)R'_{sh}(E)}{(R_{sh}(E))^2}\right) - I'_{sat}(E) \tag{52}$$

Since I_{sat} is a decreasing function in $]0, \alpha[$ (it has been proved numerically), $I'_{sat}(E) < 0$ for all $E \in]0, \alpha[$. By other hand, under (14) and (36), $R'_s(E) > 0$, $R_{sh}(E) > 0$, and $R'_{sh}(E) < 0$, for $E \in\ ]0, \alpha[$. As a consequence, the following result is obtained.

Proposition: Under (14) and (36), the photocurrent I_{ph} is increasing in $]0, \alpha[$ when $R_s(0) \geq 0$ and, as a consequence,

$$I_{ph} \in \left]I_{ph}(0), I_{ph}(\alpha)\right[\tag{53}$$

where

$$I_{ph}(0) = \lim_{E \to 0^+} I_{ph}(E) = I_{sc} - I_{sat}(0) \tag{54}$$

$$I_{ph}(\alpha) = \lim_{E \to \alpha^-} I_{ph}(E) = I_{sc}\left(1 + \frac{R_s(\alpha)}{R_{sh}(\alpha)}\right) \tag{55}$$

Fig. 7 illustrates the behaviour of I_{ph} as a function of E.

3.8. Parameters with or without physical meaning

In the previous subsections, the domain of each parameter of the SDM has been stablished without taking into account if these domains contain parameters without physical meaning, that is, negative currents, negative resistances or ideality factors less than 1 or greater than 2. As it can be observed, the domain of the parameters R_{sh}, I_{sat}, and I_{ph} only includes positive values, nevertheless, the theoretical domain of parameter R_s sometimes include negative values, concretely it occurs when $R_s(0) < 0$. By other hand, since the domain of the ideality factor is of the form $]0, n(0)[$, values smaller than 1 are always included in the domain; obviously, values greater than 2 are theoretically possible when $n(0) > 2$. Even, there exist cases in which $n(0) < 1$ which means that all the possible ideality factor values are smaller than 1. With the methodology proposed in this paper, it is easy to restrict the domains to include only parameters with physical meaning.

In relation with negative serial resistances, it has been found in the CEC database [38], 651 monocrystalline, 697 polycrystalline and 311 thin-film panels with $R_s(0) < 0$. This means that, for these panels, there exist I–V curves satisfying the conditions of the *remarkable points* although the corresponding parameters do not have physical sense. To avoid this situation, the parameters domains can be restricted to the values that ensure positive series resistances. Since R_s is an increasing continuous function in $[0, \alpha]$ with $R_s(\alpha) > 0$, there exists a unique $E_{min} \in]0, \alpha[$ such that $R_s(E_{min}) = 0$ and $R_s(E) > 0$ for all $E \in]E_{min}, \alpha[$, then, in such a case, the interval where R_s is positive is given in (56)

$$R_s \in]R_s(E_{min}), R_s(\alpha)[\tag{56}$$

To obtain E_{min}, it can be deduced from (32) that solving the equation $R_s(E) = 0$ in $]0, \alpha[$ is equivalent to solve the equation $D(E) = 1$ in $]0, \alpha[$, which simplified becomes in the equation

$$k_2 + ln\left(\frac{\alpha - E}{\beta - E}\right) + \left(\frac{\alpha + \gamma}{\beta} - 1\right)\frac{\gamma - E}{\alpha - E} = 0 \text{ for } E \in]0, \alpha[\tag{57}$$

where k_2, α, β, and γ where given in (22).

In such case, the remaining parameters must be restricted from E_{min} taking into account their monotonic properties. Specifically:

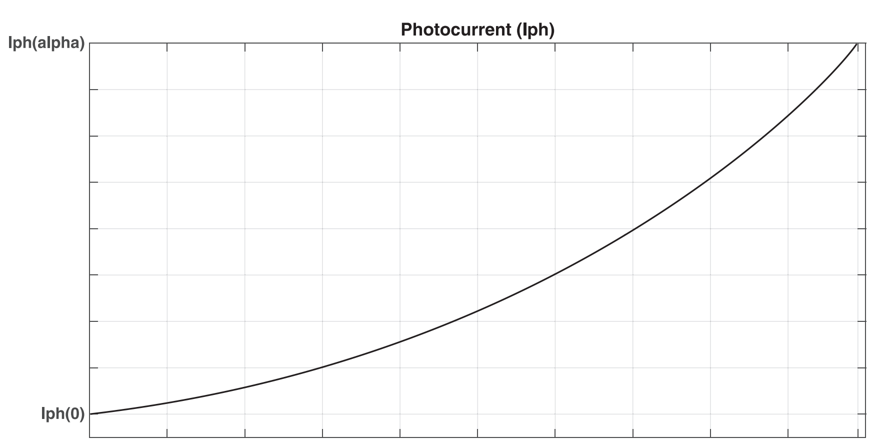

Fig. 7. The photocurrent I_{ph} in [A] as a function of E in [Ω^{-1}].

$$n \in]n(\alpha),\ n(E_{min})[\tag{58}$$

$$R_{sh} \in]R_{sh}(\alpha), R_{sh}(E_{min})[\tag{59}$$

$$I_{sat} \in\ ]0\ ,\ I_{sat}(E_{min})[\tag{60}$$

$$I_{ph} \in \Big] I_{ph}(E_{min}), I_{ph}(\alpha) \Big[\tag{61}$$

where $n(E_{min})$, $R_{sh}(E_{min})$, $I_{sat}(E_{min})$, and $I_{ph}(E_{min})$ can be directly obtained substituting E_{min} in their respective formulas (27), (40), (43) and (51).

With respect to values of the ideality factor with or without physical sense, some authors (see, for instance, Ref. [11]) have previously pointed out that, maybe, for those panels with all the possible values of the ideality factor smaller than 1 (those with $n(0) < 1$), the SDM is not suitable to study them and other models should be considered. With respect to panels with possible ideality factor values greater than 2 (those with $n(0) > 2$), the situation is not so clear because some papers (see, for instance, Ref. [39]) justify the existence of such a high values (up to 3, even 6) when the energy disorder of the transport layer is large enough in organic solar cells. So, values of the ideality factor greater than 2 could be considered as physically possible.

In any case, if the interval of the ideality factor wants to be restricted to some interval $[n_{min}, n_{max}]$, the equations $n(E) = n_{min}$ and $n(E) = n_{max}$ must be solved providing E^n_{min} and E^n_{max} (note that $E^n_{max} < E^n_{min}$), respectively, which allow to obtain the domains of the remaining parameters taking into account their monotonic properties.

4. Computation of a unique solution with additional data

Once obtained $B(E)$, $C(E)$, and $D(E)$, from (16), (17) and (18) respectively, substituting them in the equation (15) it is obtained

$$I = I_{SC} - EV - B(E)C(E)^V D(E)^I \tag{62}$$

For each $E \in]0, \alpha[$, (62) provides a I–V curve satisfying exactly the maximum power and open circuit conditions and almost exactly the short circuit one. So, just an extra data is needed to obtain E and, therefore, a unique I–V curve.

4.1. An extra point

If an extra point $(\tilde{V}, \tilde{I})$ were provided in the datasheet, in the same conditions of the remarkable datasheet points, substituting it in equation (62) and solving the resultant equation with unknown E

$$\tilde{I} = I_{SC} - E\tilde{V} - B(E)C(E)^{\tilde{V}} D(E)^{\tilde{I}} \tag{63}$$

the solution $\tilde{E}$ would be obtained and, straightforwardly, the corresponding $B(\tilde{E})$, $C(\tilde{E})$, and $D(\tilde{E})$.

4.2. A known parameter

If any parameter (I_{ph}, I_{sat}, n, R_s, R_{sh}, or A, B, C, D, E) is known, it can be imposed in the corresponding equation stated in the previous sections with unknown E. After solving the corresponding equation, we would obtain $\tilde{E}$ and the corresponding parameters $B(\tilde{E})$, $C(\tilde{E})$, and $D(\tilde{E})$, providing again a unique model solution.

4.3. The slope at short circuit point

A special case arises when the slope of the I–V curve at short circuit point, I'_{sc}, is known. Regarding that $I_{SC} \cong I_{SCE}$, the following equation is obtained

$$I'_{SC} = -E - B(E)D(E)^{I_{SC}} \left(lnC(E) + I'_{SC} lnD(E) \right) \tag{64}$$

which provides, after solving it, the value of E and, so, the determination of the model.

4.4. A fifth equation

Datasheets manufacturers not only provide the remarkable data points in different environmental conditions, usually Standard Conditions (STC) and Normal Operation Conditions (NOCT), but also the variations of the remarkable points (normally in STC) under increments of temperature and irradiance.

A big amount of papers in the literature have tried to explain the behavior of the remarkable points and also the model parameters in terms of temperature and irradiance. Some of the corresponding expressions have been used as a fifth equation to determine a unique solution of the model. Then, they try to demonstrate the validity of the obtained solution checking if the corresponding I–V curve fits accurately the datasheet data in STC and NOCT conditions, usually just for one or two specific panels because the obtained solution cannot be usually extended to other different panels. The problem is not only the truthfulness of the fifth equation but also the accuracy of the model resolution method.

The results obtained in the present paper allow, given a fifth equation, to solve the model right away just by substituting the parameters in this equation by our parameters in terms of E and, then, solving the resulting equation of the unique variable E.

An important application of our methodology, could be to check if the fifth equation is compatible with the four equations obtained from the remarkable datasheet points.

5. Experimental results

With the aim to show the practicality and effectiveness of the theoretical results provided in previous sections, some examples of application to real PV modules are presented below.

5.1. Parameters feasible domain

In Table 3, the STC datasheet data for five commercial panels are presented, these panels are the same used in Refs. [11,40], and [41]. The feasible domains of the SDM parameters have been calculated using the method proposed in the previous sections, and they are presented in Table 4. It is worth noting the panels SunPower SPR-315 and Atersa A-130 domains have been restricted just to those parameters satisfying that the series resistance is positive as explained in section 3.8.

In Fig. 8 it is depicted the feasible curves region and three arbitrary different solutions of the model corresponding to the

Table 3
Commercial panels datasheet data.

PV Module	Isc(A)	Voc(V)	Impp(A)	Vmpp(V)	Cells
Suntech STP-280	8.33	44.8	7.95	35.2	72
SunPower SPR-315	6.14	64.6	5.76	54.7	96
Atersa A-120	7.7	21	7.1	16.9	36
Atersa A-130	4.55	41.4	4	32.5	72
Isofoton I-110	3.38	43.2	3.16	34.8	72

Table 4
Feasible Domains of the SDM parameters.

PV Module	Limits	n	Rs(Ω)	Rsh(Ω)	Iph(A)	Isat(A)
STP-280	Min	0	0.6501	91.4240	8.33	0
	Max	0.7759	1.2075	Inf	8.44	2.3272E-13
SPR-315	Min	0	0	142.2286	6.14	0
	Max	1.4312	1.7187	6213.82	6.2142	6.93103E-8
A-120	Min	0	0.0817	27.5892	7.7	0
	Max	1.4909	0.5775	Inf	7.8612	1.87689E-6
A-130	Min	0	0	56.8659	4.5499	0
	Max	2.1677	2.2250	505.059	4.7280	1.46846E-4
I-110	Min	0	0.6960	155.5236	3.38	0
	Max	1.2268	2.6582	Inf	3.4378	1.83121E-8

Atersa A-120 panels, the parameters of these curves are listed in Table 5. It should be pointed out that, although the inferior extreme curve parameters are detailed, they do not provide a real SDM I–V curve.

5.2. Unique solution with additional data

As explained in section 4, a unique solution of the model can be achieved if an extra point, a known parameter, the slope at short circuit point or a fifth equation is available. In this section, five different curves have been obtained for the Sunpower SPR-315 panel using the following five different approaches proposed in the literature:

- Ideality factor equal to 1.3. This value is suggested for the ideality factor in many papers without specific analysis [37] (see also comments in Ref. [11]).
- Heuristic rule. A possibility proposed in Ref. [11] is to consider the ideality factor equals to the 90% of the maximum feasible value.
- The slope of the I–V curve at the short circuit point is given by the relation (65) in Ref. [21]. Since R_{sh} can be written as function of E, see equation (40), equation (65) can be easily solved.

$$I'_{SC} = -\frac{1}{R_{sh}} \tag{65}$$

- As proposed in Ref. [13], using the voltage temperature coefficient, α_v, and the current temperature coefficient, α_i, the ideality factor in STC, n_{STC}, is suggested by equation (66) which only depends on the remarkable datasheet points and other data in STC (denoted with STC as part of the sub-index). Once the ideality factor is calculated, the other parameters are directly computed.

$$n_{STC} = \frac{\alpha_v - \frac{V_{OC,STC}}{T_{STC}}}{n_s \cdot V_{t,STC} \cdot \left(\frac{\alpha_i}{I_{SC,STC}} - \frac{3}{T_{STC}} - \frac{E_{gap,STC}}{k \cdot T_{STC}^2} \right)} \tag{66}$$

- The fifth equation (67) proposed in Ref. [14]. In this case, the datasheet voltage temperature coefficient is used to obtain the open circuit voltage at a different temperature, T'. Since all the parameters can be written in terms of E, the equation can be solved and the parameters extracted. Although some authors state that the value of the temperature increment has low influence on the solution (see for example [14,42,43]), two different increments ($\Delta T = 5K$ and $\Delta T = 10K$) have been used in order to check if there exists a perceptible difference.

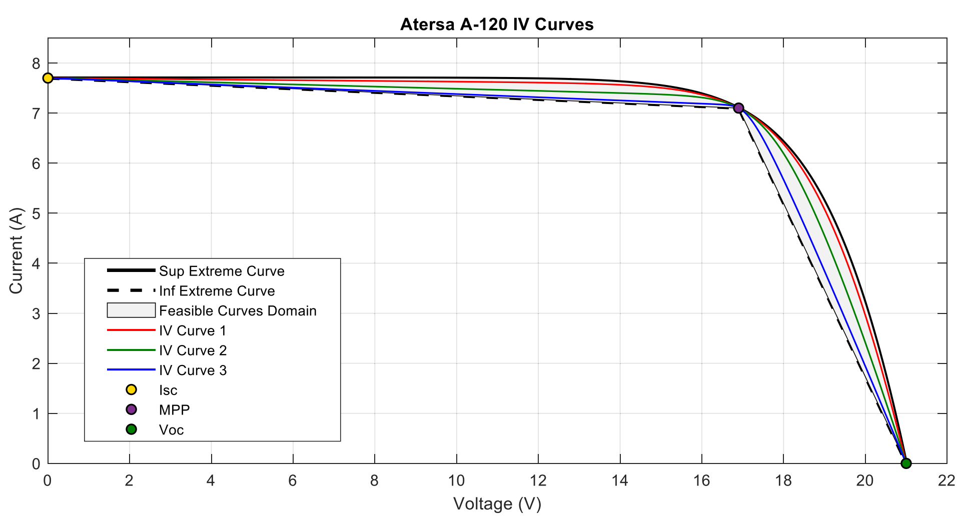

Fig. 8. Atersa A-120 panel feasible curves region, extreme curves and three feasible curves.

Table 5
SDM parameters for Atersa A-120.

Atersa A-120 Parameters					
Curve	n	Rs(Ω)	Rsh(Ω)	Iph(A)	Isat(A)
Superior Extreme	1.4909	0.0817	Inf	7.7	1.87689E-6
Inferior Extreme	0	0.5775	27.5892	7.8612	0
IV Curve 1	1.1740	0.1560	140.6774	7.7085	3.02525E-8
IV Curve 2	0.5608	0.3284	46.6160	7.7542	1.915853E-17
IV Curve 3	0.13233	0.4953	30.8009	7.8238	2.219622E-74

Table 6
Fifth equation compatibility for Sunpower SPR-315 panel.

SUNPOWER SPR-315 Parameters					
Curve	n	Rs(Ω)	Rsh(kΩ)	Iph(A)	Isat(A)
Villalba et al. [12]. Ideality Factor 1.3	1.3	0.110921	1.342357	6.140507	1.086506E-8
Laudani et al. [11] Heuristic Rule Id. Factor $0.9 \cdot n_{max}$	1.288098	0.121204	1.252904	6.140594	9.014765E-9
Chan et al. [8] Fifth Equation Slope at SCP	1.369340	5.176404e-02	2.294489	6.140138	3.023014e-08
Accarino et al. [13] Ideality Factor formula	1.020959	0.362868	0.499144	6.144464	4.358527e-11
DeSoto et al. [14] Fifth Eq. with ΔT = 5K	0.9498088	0.4311707	0.4294011	6.146165	6.359907e-12
DeSoto et al. [14] Fifth Eq. with ΔT = 10K	0.9493954	0.4315729	0.4290516	6.146176	6.283895e-12

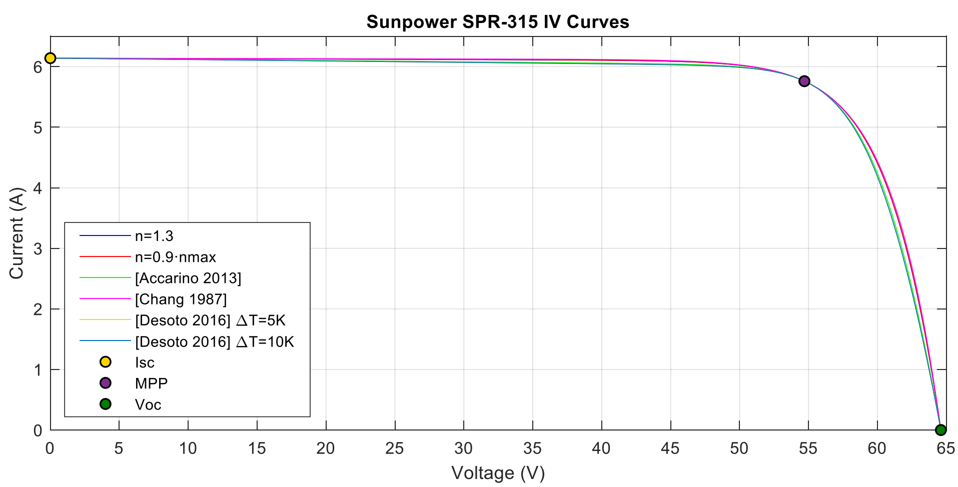

Fig. 9. SPR-315 different 5th equation curves.

$$0 = I_{ph} + \alpha_v \cdot (T' - T_{STC}) - I_{sat}\left[\frac{T'}{T_{STC}}\right]^3 e^{\left[\frac{E_{gap\,STC}}{k \cdot T_{STC}} - \frac{E_{gap}}{k \cdot T'}\right]}\left[exp\left(\frac{V_{oc}(T')}{N_s\, n_{STC}\, V_{t,T'}}\right) - 1\right] - \frac{1}{R_{sh}} \cdot V_{oc}(T') \tag{67}$$

Five Sunpower SPR-315 unique parameters solutions, using different extra datum approaches, are presented in Table 6. It should be pointed out that all the solutions are different, even the two ones using the fifth equation from Ref. [14] under two different temperature increments. The fifth equations are compatible with the remarkable datasheet points, and all the obtained parameters are inside the corresponding feasible domains (see Table 4). So, nowadays, there is not a definitive extra datum, including a fifth equation, providing the desired unique solution of the SDM. Nevertheless, although the parameters are different, all the curves in this particular case are quite similar as can be seen in Fig. 9. It is worth noticing that, as already stated in Ref. [11], there are some modules in the CEC database for which each one of the fifth equations indicated above provides a corresponding solution that is incompatible with the domain of the parameters according to exact theory previously discussed.

5.3. Estimated short circuit current error

In order to analyse the error generated by the assumption $K = I_{SC}$, the maximum difference between the estimated I_{SC} calculated using the model (which corresponds to the minimum value of E) and the I_{SC} provided in the datasheet, has been calculated for all the panels provided in the CEC database. In particular, the maximum difference has been computed for 3613 mono-Si panels, 4610 poly-Si panels and 621 thin-film panels. In the CEC database there are many more entries (more than 22000) but, as commented before, many of them are repetitions or the data are wrong or incomplete. For these reasons, for mono-Si and poly-Si panels the data provided

Table 7
IscE maximum error (%).

IscE maximum error (%)				
Technology	Number of Modules	Maximum Error (%)	Minimum Error (%)	Average Error (%)
Monocrystalline	3613	0.00200	0	0.00005
Polycrystalline	4601	0.01087	0	0.00007
Thin Film	621	0.40760	0	0.01762

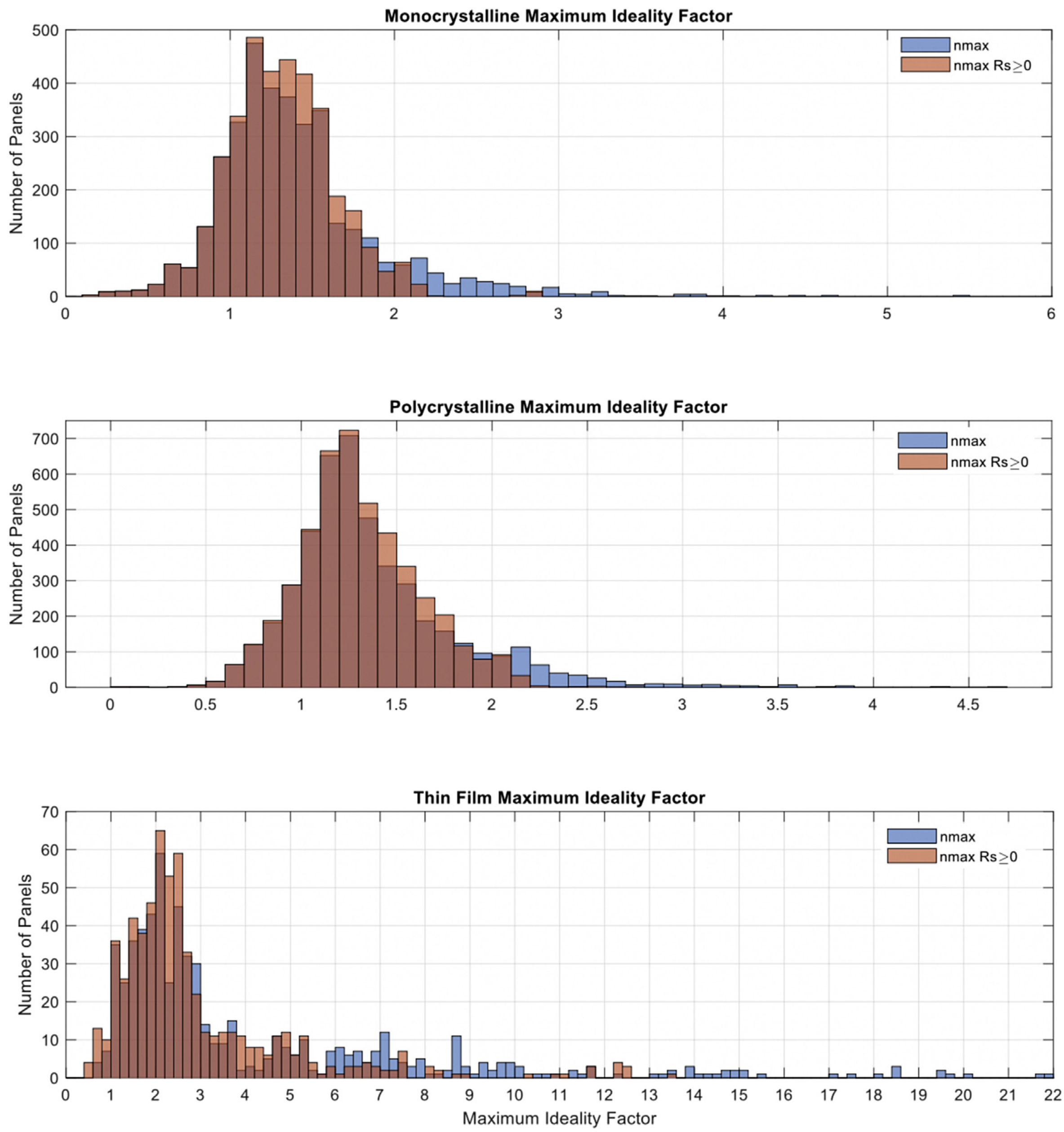

Fig. 10. Histograms of nmax values for different technologies.

in Ref. [11] additional material has been used and, for thin-film, 621 panels have been selected, discarding repetitions and with incorrect data. The domains have been restricted just to those parameters satisfying that the series resistance is positive as explained in subsection 3.8. The results are presented in Table 7. It can be seen that the maximum error for silicon modules is lower than 0.011% and, for thin film modules lower than 0.41%, but these are extreme cases, in general, the error is significantly lower as can be seen with the average error values. So, it can be concluded that the proposed hypothesis (14) and, so, the proposed model (15) and the subsequent methodology results, are almost exact.

5.4. Analysis of ideality factor feasible domain

As complement to the analysis presented in Ref. [11], a statistical analysis of the ideality factor feasible domain has been performed with the data provided in the CEC database.

In Fig. 10 it is shown the distribution of the ideality factor maximum values for the different technologies. In blue it is depicted the distribution without the $R_s \geq 0$ restriction and, in orange, with the restriction applied. It can be observed that, when the restriction is applied, maximum values of ideality factors higher than 2 are reduced because they are related with negative values of the model series resistance. The behaviour of mono-Si and poly-Si panels is very similar with most of the values located between 1 and 2. In relation with the thin-film panels, the behaviour is different, higher ideality factor maximum values are obtained although the restriction is applied. The dispersion in this type of panels is quite high and most of the values are located between 1 and 3.

The results obtained for mono-Si and poly-Si panels are almost equal as the ones presented in Ref. [11] so, the conclusions are similar:

- The adoption of fixing the ideality factor to 1.3 that is used in many papers as in Ref. [12], is not compatible with the data provided in the datasheet for many panels.
- Some panels have a maximum ideality factor lower than 1. This means that the SDM is not suitable for modelling the physical behaviour of this panels, although the theoretical generated I–V curve fits correctly.
- There are some cases with maximum ideality factor above 2. As this is the maximum value compatible with the datasheet data, but it has not physical meaning, in these cases the maximum value can be restricted to 2.

In the case of Thin-Film panels some new conclusions can be extracted from the analysis:

- The dispersion of maximum ideality factor values in this technology is quite high, although most of the results are between 1 and 5. These values are compatible with recent studies that demonstrate that the ideality factor in this type of panels can go up to 6, when the energy disorder of the transport layer is large enough [39].
- High maximum ideality factor values are associated with low fill factors and loss of V_{OC}, issues that are typical in thin film panels.
- Only few thin-film panels have maximum ideality factor lower than 1. So, the SDM could be adequate to model the physical behaviour of these type of panels.

6. Conclusions

In this paper, the domain of the five parameters corresponding to the SDM I–V curves satisfying the remarkable points conditions given in the solar datasheets, namely, short circuit, open circuit and maximum power points together with the slope at the MPP, is obtained for the first time. The main idea has been to express four of the five parameters as functions of the remaining one and, then, demonstrate that these functions are monotonic. The presented methodology allows to obtain easily and directly the possible I–V curves simply by solving a one-variable equation whenever an extra datum is provided, for example, any SDM parameter, the slope at the short circuit or at the open circuit point, or a fifth equation relating the parameters. An important hypothesis (14) duly justified have been used in the methodology but, moreover, it has been analyzed in Subsection 5.3 confirming its reliability. The applicability of the obtained results has been demonstrated in panels of different types of technologies, namely, mono-Si, poly-Si and thin-film, providing not only the intervals of the parameters but also computing the extreme I–V curves and giving some interior ones by way of illustration. Some extra data or fifth equations very well-known in the literature have also been implemented using the proposed methodology, and their validity have also been analyzed concluding that, until now, it is not possible to say that any of them is the unique representative of the corresponding panel. Finally, a complementary statistical study of the one performed in Ref. [11] for the ideality factor, has been carried out with similar conclusions for mono-Si and poly-Si panels, and new insights for the thin-film ones.

To finish, it is also interesting to note that the method proposed in the present paper, not only works for the remarkable points in STC, also works for the same points given in nominal operating conditions (NOCT) or for any other conditions whenever the remarkable points satisfy certain hypotheses stated in the results.

Any interested researcher can test the proposed method online at https://pvmodel.umh.es/ivdomain. In this webpage, just entering the datasheet remarkable points, the feasible domain of each parameter is automatically calculated.

7. Annex: step-by-step algorithm pseudocode

The following pseudocode provides the extremes of the intervals to which the five parameters of the SDM must belong to satisfy the remarkable point conditions given in the datasheets.

Datasheet data

Isc, Voc, Vmpp, Impp, Ns (Number of series cells), Temp (in Kelvin degrees)

Definitions

```
k = 1.3806503*10^(−23)
q = 1.602*10^(-19)
Vt = k*Temp/q
alfa=(Isc-Impp)/Vmpp
beta = Isc/Voc
gamma = Impp/Vmpp
k1 = 1/(2*Vmpp-Voc)
k2 = log(Vmpp/Voc)
B0 = exp(log(Isc)-(Voc/(2*Vmpp-Voc))*(log((Isc-Impp)/Isc)+Impp/(Isc-Impp)))
C0 = exp(k1*(k2+log(alfa/beta)+gamma/alfa))
```

Domain calculation

```
nmin = 0
nmax = 1/(Vt*Ns*log(C0))
Rsmin = 1/gamma*(1+((alfa + gamma)/beta-2)*(gamma/alfa)/(k2+log(alfa/beta)+gamma/alfa))
Rsmax=(Voc-Vmpp)/Impp.
Rshmin=(Vmpp/(Isc-Impp)-(Voc-Vmpp)/Impp)
Rshmax = inf
Ismin = 0.
Ismax = B0.
Iphmin = Isc-Ismax.
Iphmax = Isc*(1+Rsmax/Rshmin)
```

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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